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WORKSHOP MATHEMATICS

PART I

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WORKSHOP MATHEMATICS

PART I.

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PREFACE.

To perform his work intelligently, an artizan must have a knowledge of Elementary Mathematics. When he comes to appreciate this fact for himself the workman generally finds that even the arithmetic he learnt at school has left him, and that he remembers little more than four simple rules and the multiplication table. Teachers soon discover that though anxious to learn, a student of this kind does not wish to lose contact with the practical requirements of the workshop,—he is impatient of "pure" mathematics,—so the question arises how to teach him mathematics enough, by dealing with the calculations themselves which he is actually called upon to make at his work.

The plan which is found most successful is a compromise. It is useless to say that all students ought to learn the broad principles of mathematics first, and apply them afterwards. Experience has proved that most artizans will not attend classes where the authorities decide that this is the only course.

To meet the difficulty classes in Workshop Arithmetic, Workshop Calculations and Practical Mathematics, have grown up, and it is to provide for young workmen beginning to attend one of these classes that this little book has been prepared. It will form with its sequel an introduction to my larger volume on "Practical Mathematics" which has been received very

favourably, and will, I trust, prove serviceable to a class of students who deserve every assistance.

A long experience in my own classes has convinced me that the solution of a large number of carefully graduated exercises of a practical kind is the best way to maintain the interest of the student. It will consequently be found that the most prominent characteristic of the present book, and of the supplementary part, is the subordination of rigid mathematical proof to the provision of numerous problems drawn from the student's everyday experience.

FRANK CASTLE.

LONDON, *July*, 1900

PREFACE TO NEW EDITION.

AT the end of this edition Miscellaneous Exercises have been added, arranged in sections, corresponding roughly to those adopted in the book. It is hoped that these may be found useful either for working simultaneously with the sections to which they refer, or as revision exercises. Some corrections to the answers have been made; and this opportunity is taken to thank those teachers who have directed my attention to the need for them. Acknowledgments are due to the Union of Lancashire and Cheshire Institutes [U.L.C.I.]; The Union of Educational Institutions [U.E.I.]; The National Union of Teachers [N.U.T.]; and other authorities who have kindly permitted the use of questions from their examination papers.

F. C.

HASTINGS, *August*, 1918.

CONTENTS.

WORKSHOP MATHEMATICS.

CHAPTER I.

SIGNS OR SYMBOLS OF OPERATIONS. PRIME NUMBERS, FACTORS, POWERS, MEASURES, G.C.M., L.C.M.

THE student who is not already familiar with the symbols or signs of operations used in Arithmetic, will find they are labour-saving and very convenient. It is, therefore, advisable to become acquainted with them as soon as possible. It is hoped, however, that the majority of the symbols which will be introduced as required in the following pages are already known; if this is not so, they should be employed on all possible occasions until they can be used quickly and accurately.

Signs of Operation: Addition.—The sign + (called plus) indicates that the number before which it is placed is to be added to the number in front of it. Thus $4+5$ means that 5 is to be added to the number 4; also $4+5+7$ means that 7 is to be added to 5, and then 4 to the result, or, simply the three numbers are to be added together.

The sign = is used as an abbreviation of the words "is equal to," or simply "equals." Thus $4+5+7=16$ is read as 4 plus 5 plus 7 is equal to, or equals, 16.

Subtraction.—The sign − (called minus) is the sign of subtraction, and indicates that the number which follows it is to be

taken away, or *subtracted* from, the number which is placed before it.

Thus $12-4$ (read as 12 minus 4) means that 4 is to be subtracted from 12; hence $12-4=8$.

Multiplication.—When one number is multiplied by another the sign $\times$ is placed between the numbers; thus 6×4 means "6 multiplied by 4," or 6 times 4, or the *product* of 6 and 4.

From Fig. 1, where 4 rows of dots are arranged, each row containing 6 dots, the result of multiplying 6 by 4 can be obtained by the simple process of addition, and the process of multiplication is thus seen to be a brief and concise method of adding a given number several times to itself. Although this method can be used when the numbers are small, it would be a very troublesome process with large numbers.

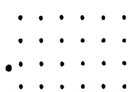

FIG. 1.—To show that $6\times4=4\times6$.

Again, as in Fig. 1, there are 6 vertical rows of dots each containing 4, the total number of dots is the product of the numbers 6 and 4, or 6×4; similarly, there are 4 horizontal rows each containing 6, and the product is 4×6; hence $4\times6=6\times4$.

The number 6 preceding the sign of multiplication, $\times$, is called the **multiplicand**; the number 4 following it is called the **multiplier**; and, as has been seen, the final result is called the **product**. Moreover, since $6\times4=4\times6$, the multiplicand and multiplier are seen to be interchangeable. The two numbers themselves are called **factors** of the number denoting the product.

Division may be indicated by placing the symbol $\div$ between two or more quantities. Thus, $8\div2=4$ (which is read as 8 divided by 2 is equal to 4; or, 2 into 8 is 4). Such division is often expressed by writing the two numbers in a fractional form, as $\frac{8}{2}=4$.

The number preceding the sign of division, $\div$, is called the **dividend**; the number following it is called the **divisor**; and the result, when the operation indicated by the sign is performed, is called the **quotient**.

In this way it is seen that the signs or symbols of operations

give definite information, enabling operations and results to be written in a brief and clear manner.

All these operations may be summarised by the following simple examples :

$$4+5=9; \qquad 12-4=8;$$
$$5\times 6=30; \quad 8\div 2, \text{ or } \tfrac{8}{2}=4.$$

Prime Number.—Any number which is divisible by no other number except unity is called a prime number ; thus 1, 3, 5, etc., are prime numbers.

Measure, Multiple, or Factor.—A number which divides an exact number of times into another number, is said to be a *measure* or *factor* of the latter; it is also called a *sub-multiple* of that number.

$$14=7\times 2.$$

Either 2 or 7 will divide the given number 14 without a remainder, hence either of these numbers is said to be a *measure* of 14.

$$21\div 3=7.$$

Since 3 divides into 21 without a remainder it is said to be a *measure* of 21.

These examples suggest a rule which may be stated thus :

RULE.—*If in the division of one number by another there is no remainder, the divisor is a measure of the dividend.*

Factors.—The two numbers 2 and 7, or 3 and 7 in the above examples, are both said to be factors of 14 and 21, because when multiplied together they give the numbers 14 and 21 respectively.

Squares and Cubes. Powers.—When a number is multiplied by itself the result is called the *square* or the *second power* of the number. Thus the square of 3, or 3×3, is 9, and the square, or second power, of 4 is 4×4, that is, 16.

When three numbers of the same kind are multiplied together the result is called the *cube* or *third power* of the number ; thus, the cube of 3 is $3\times 3\times 3$, that is, 27. The cube of 4 is $4\times 4\times 4$ or 64.

In the same manner, the result of multiplying four numbers of the same kind together is called the *fourth power*. Thus, the fourth power of 4 is $4\times 4\times 4\times 4$. that is, 256.

By multiplying five of the same numbers together we should obtain the fifth power of the number. The sixth, seventh, or any other power, of a number is determined in a similar manner.

Index.—Another and a very convenient method of indicating the power of a number is by means of a small figure placed near the top and on the right-hand side of a figure; thus the square or second power of 2 may be written 2×2, but more conveniently as 2^2, and the fourth power of 2 either as $2 \times 2 \times 2 \times 2$, or 2^4. Similarly the square, cube, fourth or fifth power of 4 would be written 4^2, 4^3, 4^4 and 4^5.

Greatest Common Measure.—In addition to the measures just referred to, another important measure of two or more numbers is called the Greatest Common Measure of those numbers, and may be defined as follows:

RULE.—*The greatest number which is contained an exact number of times in each of two or more given numbers is called their Greatest Common Measure*, or, as usually written, G.C.M.

One method which may be used to find the G.C.M. of two or more numbers is to break them up into factors.

Thus, to find the G.C.M. of the two numbers 14 and 21. The greatest number which is contained an exact number of times in the two numbers is the number 7, hence 7 is the G.C.M. required.

To find the G.C.M. of the two numbers 45 and 126.

On breaking both numbers into factors, we have

$$45 = 9 \times 5,$$

$$126 = 9 \times 7 \times 2,$$

and the G.C.M. $= 9$.

Ex. 1. Find the G.C.M. of 21879 and 14872.

Since $21879 = 9 \times 11 \times 13 \times 17,$

and $14872 = 8 \times 11 \times 13 \times 13.$

The G.C.M. is $11 \times 13 = 143$.

Ex. 2. To find the G.C.M. of 2079 and 2898.

Since $2079 = 9 \times 3 \times 7 \times 11,$

and $2898 = 9 \times 2 \times 7 \times 23.$

The G.C.M. $= 9 \times 7 = 63$.

When the numbers are large, the process of finding the factors is rather tedious, and it is better to use the following process:

RULE.—To find the G.C.M. of two numbers:—*Divide the larger of the two numbers by the smaller; then divide the smaller number by the remainder from the first division; divide the first remainder by the new remainder, if any, and so on until there is no remainder. The last divisor is the G.C.M. required.*

Thus, to find the G.C.M. of 21879 and 14872.

```
14872 ) 21879 ( 1
        14872
         7007 ) 14872 ( 2
                14014
                  858 ) 7007 ( 8
                        6864
                         143 ) 858 ( 6
                               858
```

∴ G.C.M. = 143.

In finding the G.C.M. of two or more numbers, the work is often simplified by taking out any common factor of the numbers before the division is made. This common factor must, however, be included in the G.C.M.

When the G.C.M. of more than two numbers is required, the G.C.M. of two may first be found, and afterwards the G.C.M. of this first result and the third number, and so on.

Thus, to find the G.C.M. of 3024, 4752, and 7488.

The G.C.M. of 3024 and 4752 found as in the last worked out example is 432. In like manner the G.C.M. of 432 and the remaining number 7488 is found to be 144. Hence, 144 is the G.C.M. of the three given numbers 3024, 4752 and 7488.

EXERCISES. I.

1. Find the highest number that will divide both 12499 and 14790 without a remainder. What is that highest number called?

2. What is meant by the Greatest Common Measure of two numbers?

Find the G.C.M. of the following sets of numbers:

3. 306, 357 and 504.	**4.** 420 and 480.
5. 5777 and 7261.	**6.** 40991 and 48151.
7. 23760 and 26136.	**8.** 6023 and 15466.
9. 14938, 23474 and 32010.	**10.** 116039, 122067 and 137137.
11. 7560, 27720 and 108108.	**12.** 15496 and 12665.
13. 10058, 4982 and 9823.	**14.** 8775 and 12025.
15. 27531 and 8740.	**16.** 2301 and 3717.
17. 4781 and 6147.	**18.** 7535, 11645 and 6165.
19. 2622, 2793 and 2736.	**20.** 282660 and 40299.
21. 1019527 and 1231845.	**22.** 75582, 42237 and 5103.
23. 24057 and 3645.	**24.** 3024, 4752 and 7488.
25. 17255 and 14161.	**26.** 13536 and 23148.

Least Common Multiple.—*When a number contains another number an exact number of times, the first number is called a multiple of the second.* Thus, 15 is a multiple of 3 and 5; 12 is a multiple of 3, 4, and 6.

The smallest number which is divisible by each of two or more given numbers is called the Least Common Multiple of the given numbers, and is usually written L.C.M.

One method which may be used to find the L.C.M. of two or more given numbers is to split up the numbers into their prime factors.

Having obtained the factors, select the highest powers of each prime factor which occurs in these products; by multiplying these highest powers together the L.C.M. is obtained.

Ex 1. Find the L.C.M. of 42, 56 and 63.

Here
$$42 = 3 \times 7 \times 2,$$
$$56 = 8 \times 7 = 2^3 \times 7,$$
$$63 = 9 \times 7 = 3^2 \times 7.$$

The highest powers of 2, 3, and 7 that occur are 2^3, 3^2 and 7.

Hence the L.C.M. $= 2^3 \times 3^2 \times 7$
$$= 8 \times 9 \times 7 = 504.$$

A very convenient and expeditious method of finding the prime factors of numbers, the L.C.M. of which is required, is as follows.

Arrange the given numbers in line, separating them by commas

Divide the numbers by any factor which is a factor of two or more of them.

Thus, in solving the above example by this method, first, dividing by 2, next by 3, and finally by 7, we obtain in the last line the numbers 1, 4, 3.

$$\begin{array}{r|lll} 2 & 42, & 56, & 63 \\ \hline 3 & 21, & 28, & 63 \\ \hline 7 & 7, & 28, & 21 \\ \hline & 1, & 4, & 3 \end{array}$$

The L.C.M. is the product of the numbers in the last line and the numbers which have been used as divisors.

$$\therefore \text{ L.C.M.} = 2 \times 3 \times 7 \times 4 \times 3$$
$$= 504.$$

When the given numbers are large, a similar method of arranging the numbers in line may be employed, but the G.C.M. of the numbers must be used as the divisor.

Ex. 2. Find the L.C.M. of 3024, 4752 and 7488.

In this case the G.C.M. as on p. 5 is found to be 144. Dividing by the G.C.M. we obtain:

$$\begin{array}{r|lll} 144 & 3024, & 4752, & 7488 \\ \hline 3 & 21, & 33, & 52 \\ \hline & 7, & 11, & 52 \end{array}$$

$$\therefore \text{ L.C.M.} = 144 \times 3 \times 7 \times 11 \times 52 = 1729728.$$

Referring to the preceding example, it will be evident that the process adopted in both examples is the same. In the first of the above examples 7 is the G.C.M. of the three numbers 42, 56 and 63, and dividing by the G.C.M. the factors are 2^3 and 3^2.

The G.C.M. of the two numbers contains all the prime numbers and powers of prime numbers common to the two numbers. Hence to find the L.C.M. we have the rule: Divide the product of the numbers by their G.C.M.

In applying this rule the convenient method when two numbers are given is to *divide the G.C.M. into one of the numbers and multiply the quotient by the other number.*

Ex. 3. Find the L.C.M. of 2145 and 1287.

In the usual way we find the G.C.M. $=429$.

Hence L.C.M. $= \frac{1287}{429} \times 2145 = 6435$.

As the L.C.M. may be obtained by dividing the product of two numbers by their G.C.M., the product of two numbers is equal to the G.C.M. of the numbers multiplied by their L.C.M.

This can easily be understood by referring to Ex. 3, or by writing the problem as an ordinary division sum, thus:

G.C.M.) Product of two numbers (L.C.M.

To prove a division sum the dividend is multiplied by the divisor, hence it is seen that

G.C.M. × L.C.M. = Product of two numbers.

An old rule for finding the L.C.M. of three or more numbers may in some cases be used with advantage. The rule is: Find the L.C.M. of two of the numbers, then proceed to find the L.C.M. of the third number and the L.C.M. already obtained, and so on.

Ex. 4. Find the L.C.M. of 42, 56, and 63.

The G.C.M. of 42 and 56 is 14. The L.C.M. is $\frac{42}{14} \times 56 = 168$.

The G.C.M. of 168 and 63 is 21.

$\therefore$ L.C.M. is $\frac{168}{21} \times 63 = 504$.

Ex. 5. Find the G.C.M. and L.C.M. of 1920 and 756.

$1920 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7.$

The G.C.M. is the product of the common factors

$2 \times 2 \times 3 = 12.$

The L.C.M. $= 1920 \times 756 \div 12 = 120960.$

EXERCISES. II.

Find the Least Common Multiples of

1. 15, 20, 24, 30.
2. 44, 48, 96, 52.
3. 90, 66, 42, 30, 11.
4. 125, 27, 24, 180, 136.
5. 165, 198, 270.
6. 192, 204, 272.
7. 3024, 4752, 7488.
8. 132, 165, 220.
9. 385, 231, 165, 105.
10. 555, 1221, 2035.
11. 4781, 6147.
12. 7535, 11645.
13. 5713, 5771, 6467.
14. 78, 84, 90.
15. 132, 161, 168, 253.
16. When is one number prime to another? Find the G.C.M. and L.C.M. of 7560, 27720 and 108108.
17. State the arithmetical rule for finding the Least Common Multiple of three or more numbers. Find the smallest number which when divided by 12, 15 and 18 gives remainder 1 in each case.
18. What is the L.C.M. of two numbers? Find the least sum of money that can be exactly distributed either in half-crowns or half guineas.

SUMMARY.

19. Resolve 2310, 6552 and 2165 into their prime factors and thence deduce their L.C.M.

Find the L.C.M. of

20. 40, 42, 44, 48. **21.** 198, 495, 2475. **22.** 18, 51, 34, 7, 30.

23. Find the G.C.M. and the L.C.M. of the numbers 15496, 12665.

24. Find the G.C.M. and L.C.M. of 123, 147.

25. Find the L.C.M. of 959, 6973, 2329.

26. Find the G.C.M. of 27781, 23507.

27. Find the L.C.M. of 20, 12, 15, 18, 4; and the G.C.M. of 702, 1368, 1944.

Summary.

The signs of operation are + (addition), − (subtraction), × (multiplication), ÷ (division), and = (equals, or equal to).

Prime numbers are those numbers which are divisible by no other number except unity.

Measure or Factor. A number contained an exact number of times in one or more numbers is said to be a measure or factor of those numbers.

Powers.—When a number is multiplied once by itself the result is called the square or second power of the number. When multiplied twice by itself the product is called the cube of the number.

Index.—A small number near the top and on the right of a given number is called the index, or power of the number. Thus the square, cube, and fourth power of 2 would be indicated by 2^2, 2^3, and 2^4 respectively.

Greatest Common Measure. The greatest number which is contained an exact number of times in each of two or more given numbers is called their G.C.M. It is often also called the Highest Common Factor, or H.C.F.

Least Common Multiple.—The smallest number which contains each of two or more numbers is called the Least Common Multiple, or L.C.M., of the numbers.

CHAPTER II.

VULGAR FRACTIONS.

Fractions.—Suppose that any unit (such as a rod one yard long) is divided into three equal parts; each of these parts is called one-third ($\frac{1}{3}$) of the whole, and two of these parts together make two-thirds ($\frac{2}{3}$) Each of these quantities, $\frac{1}{3}$, $\frac{2}{3}$, is called a fraction. A *fraction* may thus be defined as a part or parts of a whole.

The section of Arithmetic dealing with the subject of fractions is very important, and should receive the best attention of the student. In Fig. 2 one end of an ordinary foot-rule ($\frac{1}{3}$ yard) is shown. The distance between *a* and *b* or *b* and *e* (which represents an inch in each case) is divided into 8 or 10, and on some

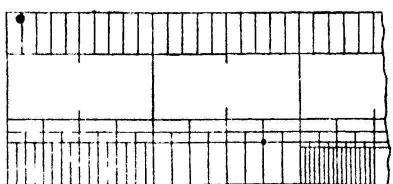

FIG. 2.—Showing one end of a scale of inches and fractions of an inch.

scales into 12 or 16 equal parts. Such a rule or scale, which every student is supposed to possess, is a useful instrument with which to illustrate the four operations of Addition, Subtraction, Multiplication, and Division. Consider any length on your scale,

such as from b to f, the third division towards e. This length is spoken of as three-eighths of an inch, and is written $\frac{3}{8}''$. The number below the line is called the **denominator** of the fraction, and indicates the number of parts into which the unit has been divided. The upper number of the fraction, in this case 3, is called the **numerator** of the fraction, and expresses the number of parts, in this case eighths, which have been taken. Or, expressed in another way,

$$\text{Fraction} = \frac{\text{numerator}}{\text{denominator}}$$

When the numerator and denominator of a fraction are each multiplied or divided by the same number, the value of the fraction is unaltered. Thus, for example, if we multiply both the numerator and the denominator of our fraction $\frac{3}{8}$ by 2, we obtain as a result the fraction $\frac{6}{16}$. This simply indicates that the unit ab (Fig. 2) is divided into sixteen parts instead of eight. Each part is therefore twice as small as before, and in order to obtain a fraction equal to the original fraction, twice as many of these smaller parts must be taken, *i.e.* 6.

By such simple methods as those suggested, it should be possible to easily understand the above propositions.

Addition, Subtraction, and Comparison of Fractions.— To add, subtract, or compare fractions having the same denominators, it is only necessary to add, subtract, or compare the numerators of such fractions. When the denominators are not alike the fractions under consideration must first be reduced to equivalent fractions, each having the same denominator.

Thus, to add $\frac{7}{20}$ and $\frac{6}{20}$ of £1 together, since the 7 and the 6, in the numerators of the fractions, indicate the number of shillings concerned, the answer is 13 shillings or $\frac{13}{20}$ of a pound; by subtracting one from the other we should get $\frac{1}{20}$ of £1.

If we wish to add $\frac{7}{8}$ and $\frac{3}{16}$ together we cannot proceed in the same way because there are 7 things of one kind and 3 of another. We must proceed to make the fractions represent things which are of the same kind, and when this is done the addition of the two fractions can be effected. Hence, we first make $\frac{7}{8}$ into $\frac{14}{16}$ by multiplying numerator and denominator

by 2, then by adding the numerators of the fractions we obtain $\frac{11}{16}$. This simple example should be verified by the foot-rule.

Improper Fractions.—Those fractions in which the numerator is not less than the denominator are called *Improper* fractions.

Any whole number can always be expressed in a fractional form, thus $6 = \frac{6}{1} = \frac{12}{2} = \frac{24}{4}$, etc.

A **Mixed Number** is a number consisting partly of a whole number and partly of a fraction; such numbers as $3\frac{1}{2}$, $5\frac{1}{3}$, $6\frac{1}{9}$, etc., are called mixed numbers.

The mixed number $6\frac{1}{9}$ indicates $6 + \frac{1}{9}$, and to represent this in a fractional form, 6 is expressed as a fraction with denominator 9, thus $\frac{6}{1} = \frac{54}{9}$; to this add $\frac{1}{9}$ and we obtain $6\frac{1}{9} = \frac{55}{9}$.

In a similar manner any mixed number may be expressed as an improper fraction.

Much trouble is frequently avoided in the addition and subtraction of mixed numbers by adding or subtracting the whole numbers first. In multiplication and division it is always advisable however to first reduce to improper fractions.

Ex. 1. Find the value of $8\frac{1}{3} + \frac{9}{5} + \frac{4}{7} - \frac{74}{105}$.

As $\frac{9}{5} = 1\frac{4}{5}$ we have:

$$\begin{aligned} 8\tfrac{1}{3} + 1\tfrac{4}{5} + \tfrac{4}{7} - \tfrac{74}{105} &= 9 + \tfrac{1}{3} + \tfrac{4}{5} + \tfrac{4}{7} - \tfrac{74}{105} \\ &= 9 + \tfrac{35}{105} + \tfrac{84}{105} + \tfrac{60}{105} - \tfrac{74}{105} \\ &= 9 + \tfrac{179 - 74}{105} \\ &= 9 + \tfrac{105}{105} \\ &= 10. \end{aligned}$$

Ex. 2. Find the value of $\frac{5}{6} - \frac{1}{8} + \frac{5}{24} - \frac{3}{16} - \frac{2}{3}$.

L.C.M. of the denominators is 48.

Hence the given fractions may be expressed

$$\tfrac{40}{48} - \tfrac{6}{48} + \tfrac{10}{48} - \tfrac{9}{48} - \tfrac{32}{48} = \tfrac{3}{48} = \tfrac{1}{16}.$$

Ex. 3. Find the value of $\frac{1}{2} + \frac{2}{3} - \frac{1}{6} + \frac{3}{8} - \frac{1}{12} - \frac{7}{24}$.

The given fractions, all with a denominator 24, are equivalent to the following,

$$\tfrac{12}{24} + \tfrac{16}{24} - \tfrac{4}{24} + \tfrac{9}{24} - \tfrac{2}{24} - \tfrac{7}{24},$$

or better written as

$$\frac{12 + 16 - 4 + 9 - 2 - 7}{24} = \frac{24}{24} = 1.$$

The arithmetical work involved in the addition, subtraction, or comparison of fractions, is in practice often considerably lessened by dealing with two or three of the given fractions at a time. When this has been done, and the fraction in its lowest terms representing the sum or difference of two or three of the terms has been obtained, proceed to add or subtract the remaining quantities.

Ex. 4. Find the value of

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{7}{6}-\frac{5}{8}-\frac{1}{10}+\frac{10}{12}.$$

Here $$\frac{1}{2}+\frac{1}{4}-\frac{5}{8}=\frac{1}{8},$$

also $$\frac{10}{12}=\frac{5}{6}, \text{ and } \frac{5}{6}+\frac{1}{3}-\frac{7}{6}=0.$$

The given fraction thus becomes

$$\frac{1}{8}-\frac{1}{10} \text{ and } \frac{1}{4}-\frac{1}{10}=\frac{1}{40}.$$

$$\therefore \frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{7}{6}-\frac{5}{8}-\frac{1}{10}+\frac{10}{12}=\frac{1}{40}.$$

In the last example, it will be noticed that by dealing with the fractions, three at a time, the arithmetical work is less than would be necessary if the L.C.M. (120) of all the denominators had been used.

EXERCISES. III.

Subtract

1. $25\frac{17}{19}$ from $31\frac{11}{76}$. 2. $\frac{71}{72}$ from $2\frac{17}{18}$.

3. $23\frac{29}{30}$ from $32\frac{33}{35}$. 4. $15\frac{29}{36}$ from $17\frac{20}{27}$.

5. Prove from the definition of a fraction that $\frac{1}{3}-\frac{1}{12}=\frac{1}{4}$. State in words the rule that you deduce for the addition or subtraction of fractions with unlike denominators.

6. Find the value of $\frac{1}{2}+\frac{2}{9}-\frac{3}{11}+\frac{7}{18}-\frac{17}{33}-\frac{32}{99}$.

7. Find the greater of $\frac{56}{143}$ and $\frac{72}{187}$, and divide $\frac{16}{11}$ by their difference.

8. Add together $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{12}$. Draw a figure to illustrate the manner in which the difficulty of the addition of fractions with unlike denominators is surmounted.

9. Find a number such that, if one twenty fourth part of it be taken away, the remainder is 253.

10. Arrange in order of magnitude the fractions $\frac{15}{16}$, $\frac{3}{4}$, $\frac{31}{32}$, $\frac{7}{8}$.

11. Add together $2\frac{1}{5}$, $3\frac{2}{9}$, $5\frac{3}{8}$. Explain why fractions are reduced to a common denominator in addition.

12. What is meant by (*a*) the numerator, (*b*) the denominator of a fraction? Prove that $\frac{3}{4}=\frac{6}{8}$, and add together $\frac{3}{10}+\frac{7}{12}+\frac{6}{24}$.

Add together

13. $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{7}{24}$.
14. $2\frac{5}{8}, 1\frac{3}{11}, 5\frac{1}{24}$ and $\frac{7}{18}$
15. $5\frac{3}{20}, 2\frac{6}{17}, \frac{13}{30}$ and $\frac{15}{34}$.
16. $5\frac{3}{10}, 3\frac{7}{24}, 5\frac{3}{8}$ and $\frac{14}{15}$
17. $3\frac{5}{9}, \frac{11}{12}, 1\frac{2}{3}$ and $\frac{1}{4}$.
18. $2\frac{7}{8}, \frac{13}{14}, 5\frac{3}{7}$ and $\frac{19}{24}$.
19. $4\frac{7}{12}, 2\frac{15}{22}, 3\frac{4}{5}$ and $\frac{9}{11}$.
20. $2\frac{2}{3}, \frac{9}{14}, 3\frac{10}{21}$ and $\frac{3}{8}$.
21. $1\frac{17}{20}, 2\frac{19}{24}, \frac{23}{36}$ and $1\frac{5}{18}$.
22. $4\frac{11}{13}, \frac{7}{8}, 1\frac{41}{52}$ and $\frac{7}{16}$.

23. What is the difference between the sum of $\frac{5}{6} + \frac{6}{7}$ and the sum of $\frac{4}{5} + \frac{5}{6}$?

24. Find the sum of $4\frac{7}{20}, 3\frac{8}{15}, 1\frac{5}{12}$.

25. What number added to $\frac{27}{32} + \frac{15}{64}$ will make $3\frac{9}{24}$?

Find the difference between

26. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ and $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$.

27. $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7}$ and $\frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8}$.

Find the value of

28. $\frac{1}{2} + \frac{2}{3} - \frac{13}{6} + \frac{3}{4} - \frac{1}{20} + \frac{4}{5}$.
29. $\frac{2}{5} + \frac{9}{10} + \frac{7}{12} + \frac{2}{15} + \frac{17}{30} - \frac{7}{12}$.
30. $\frac{1}{5} + \frac{3}{7} + \frac{1}{3} + \frac{1}{105} - \frac{1}{9}$.
31. Subtract $\frac{19}{33}$ from $\frac{4}{11}$ of $2\frac{5}{14}$.

Multiplication of Fractions.—*To multiply a fraction by any whole number, either multiply its numerator by the number, or divide its denominator by the number.* Thus, suppose the fraction is $\frac{3}{8}$ and we require to multiply this by 4. As already explained $\frac{3}{8}$ means that the unit is divided into 8 equal parts and 3 of these equal parts are taken. But 3 such parts multiplied by 4 will give 12 parts, that is, 12 eighths of the unit,

$$\frac{3}{8} \times 4 = \frac{12}{8}.$$

It is easy to prove this by reference to the scale of inches (Fig. 2), and also to show that the result would be the same if, instead of multiplying the numerator by 4, the denominator were divided by that number.

Again, $\frac{12}{8}$ reduced to its lowest terms by dividing both numerator and denominator by 4, becomes $\frac{3}{2}$; but this result would also be obtained if the denominator of the fraction $\frac{3}{8}$ were divided by 4.

It will also be obvious that the process of multiplying a fraction by any whole number is the same as adding together a corresponding number of fractions, the denominators being the same in each case.

$$\frac{7}{21} \times 3 = \frac{7}{21} + \frac{7}{21} + \frac{7}{21}, \text{ or } \frac{21}{21} = 1.$$

The result, which may be obtained either by multiplying the numerator of the fraction by 3, or dividing the denominator by that number, is the same.

Division of Fractions.—*To divide a fraction by a whole number, either divide the numerator by the number, or multiply the denominator by it.*

To divide the fraction $\frac{3}{8}$ by 4.

As shown on p. 11, $\frac{3}{8} = \frac{12}{32}$, the latter fraction meaning that the unit is divided into 32 equal parts and 12 of these parts are taken; dividing these 12 parts by 4 we obtain three such parts;

$$\tfrac{3}{8} \div 4 = \tfrac{3}{32} :$$

This again should be verified by means of a foot-rule.

Divide $\frac{12}{13}$ by 4.

Dividing the numerator by 4 we obtain the fraction $\frac{3}{13}$, or, multiplying the denominator by 4, the fraction becomes $\frac{12}{52}$, and this when reduced to its lowest terms gives the same value as before.

To multiply a fraction by a fraction.—The rule is: *Multiply the numerators together to obtain the numerator of the product, and the denominators to obtain the denominator of such product.* This can be shown to be true in any particular case in the following way:

$\frac{3}{8} \times \frac{1}{2}$, or as it is sometimes written $\frac{3}{8}$ of $\frac{1}{2}$.

Using the scale of inches (Fig. 2), as the distance from *a* to *b* is the unit, the distance *ac* will be represented by $\frac{1}{2}$. But we are directed to take $\frac{3}{8}$ of this distance, and $\frac{3}{8}$ of this half unit is $\frac{3}{16}$ of the whole distance; hence $\frac{3}{8}$ of $\frac{1}{2} = \frac{3}{16}$.

To show that $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15}$.

This may be done in a similar way to the above, or as follows:

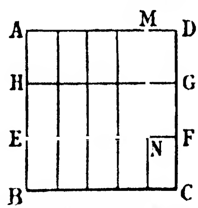

FIG. 3.—To show that $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15}$.

Construct a square *ABCD* of side equal to 5 units of length (Fig. 3). Divide the base *BC* into 5 equal parts and through the points of division draw lines at right angles to *BC* as shown. Divide *AB* into 3 equal parts. Then *AMNE* is $\frac{4}{5}$ of *ADFE*. But *ADFE* is $\frac{2}{3}$ of the square *ADCB*. Hence *AMNE* is $\frac{4}{5}$ of $\frac{2}{3}$ of the square

ABCD. But from the figure, since the square is divided into 15 equal parts, and *AMNE* contains 8 of these parts, hence

$$AMNE = \tfrac{8}{15}\,ADCB,$$
$$\therefore \tfrac{4}{5} \text{ of } \tfrac{2}{3} = \tfrac{8}{15}.$$

When three or more fractions have to be multiplied together the product of all the numerators and of all the denominators is obtained, these results are used as the numerator and denominator respectively of the required fraction.

$$\tfrac{24}{17} \times \tfrac{17}{7} \times \tfrac{7}{4} = \tfrac{24}{4} = 6.$$

Cancelling. In the above example, by *cancelling* common factors the work is performed in a simpler manner than by attempting to multiply all the numerators together to form a new numerator, and the denominators together to form a new denominator. Wherever possible it is advisable to write down all the terms, and before multiplying or dividing to cancel the common terms in both numerator and denominator.

Division of a fraction by a fraction.—*To divide one fraction by another invert the divisor and multiply the other fraction by it.* Since division is the reverse of multiplication, $\frac{3}{4}$ divided by $\frac{2}{8}$ (written $\frac{3}{4} : \frac{2}{8}$), means such a fraction which when multiplied by $\frac{2}{8}$ will give as a result $\frac{3}{4}$, and the number is $\dfrac{3 \times 8}{4 \times 2} = \dfrac{3}{1}$, or 3.

Simplification of fractional expressions.—The simplification of fractions, although in some cases rather tedious, is of great importance. The following examples will serve to illustrate some of the methods adopted:

$$\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4}.$$

Here the equivalent fractions become $\frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12}$, and the sum $\frac{13}{12}$ is obtained by adding the numerators.

Ex. 1. Which of the two fractions $\frac{5}{7}$ and $\frac{3}{5}$ is the greater?
The first is equal to $\frac{25}{35}$, the second to $\frac{21}{35}$.
Hence the first is the greater.

Ex. 2. Find the value of $\frac{5}{9}$ of $6\frac{3}{4}$ of $\frac{16}{17}$.
Here, $\frac{5}{9}$ of $6\frac{3}{4}$ of $\frac{16}{17} = \frac{5}{9} \times \frac{27}{4} \times \frac{16}{17}$
$$= \tfrac{60}{17} = 3\tfrac{9}{17}.$$

Ex. 3. Simplify $\dfrac{\frac{1}{5}-\frac{1}{6}+\frac{1}{9}-\frac{1}{18}}{\frac{1}{4}-\frac{1}{5}+\frac{1}{8}-\frac{1}{9}+\frac{1}{16}} = \dfrac{\frac{6}{30}-\frac{5}{30}+\frac{2}{18}-\frac{1}{18}}{\frac{5}{20}-\frac{4}{20}+\frac{9}{72}-\frac{8}{72}+\frac{1}{16}}$

$$= \frac{\frac{1}{30}+\frac{1}{18}}{\frac{1}{20}+\frac{1}{72}+\frac{1}{16}} = \frac{\frac{3+5}{90}}{\frac{36+10+45}{720}}$$

$$= \frac{720\times 8}{90\times 91} = \frac{64}{91}.$$

Ex. 4. Simplify $\dfrac{\frac{9}{11}\text{ of }\left(\frac{41}{162}-\frac{9}{49}-\frac{3}{54}\right)}{\frac{4}{9}+\frac{1}{2}-\frac{13}{14}}\text{ of }\left(4\frac{2}{5}-2\frac{1}{2}+\frac{3}{10}\right).$

$$\frac{\frac{9}{11}\times\left(\frac{41}{162}-\frac{9}{162}-\frac{9}{49}\right)}{\frac{17}{18}-\frac{13}{14}}\times\left(2\frac{7}{10}-\frac{1}{2}\right)$$

$$= \frac{\frac{9}{11}\times\left(\frac{16}{81}-\frac{9}{49}\right)}{\frac{119-117}{7\times 18}}\times\frac{11}{5}$$

$$= \frac{\frac{9}{5}\times\left(\frac{16}{81}-\frac{9}{49}\right)\times 7\times 18}{2} = 1\frac{4}{7}.$$

EXERCISES. IV.

1. Explain what is meant by the statement that $\frac{5}{8}$ is three times as great as $\frac{5}{24}$; also give a definition of multiplication which shall include the multiplication of one fraction by another.

2. State and explain the rule for the division of fractions. Divide $33\frac{3}{7}$ by $7\frac{34}{35}$, and bring the result to its lowest terms.

3. Find the quotient of $\frac{5}{7}$ divided by $1\frac{1}{14}$, and divide the result by $\frac{1}{30}$.

4. Multiply together $8\frac{3}{14}$ of $1\frac{9}{11}$, $\frac{16}{23}$ and $3\frac{1}{9}$ of $11\frac{19}{40}$.

5. Divide $10\frac{13}{48}$ by $6\frac{6}{29}$ and the result by $5\frac{13}{96}$.

6. Divide the difference between $4\frac{25}{36}$ and $1\frac{57}{64}$ by the sum of $2\frac{1}{6}$ and $1\frac{3}{8}$.

7. (i) Multiply together $1\frac{1}{5}$, $3\frac{1}{8}$, $1\frac{7}{9}$, and $\frac{3}{25}$. (ii) Divide $2\frac{3}{14}$ by $8\frac{6}{7}$.

8. Divide $2\frac{2}{15}+2\frac{27}{35}-3\frac{4}{21}$ by $2\frac{5}{21}+3\frac{1}{3}-4\frac{6}{7}$.

Simplify

9. $\dfrac{\frac{2}{5}\text{ of }5}{\frac{3}{8}\text{ of }1\frac{4}{5}}-\frac{23}{27}$.

10. $1\frac{1}{2}$ of $2\frac{2}{3}$ of $3\frac{3}{4}$ of $4\frac{4}{5}$.

11. $\frac{1}{2}+\frac{4}{9}-\frac{5}{11}+\frac{7}{18}-\frac{17}{33}$.

12. $2\frac{6}{7}$ of $13\frac{3}{5}$ of $7\frac{7}{10}-\frac{1}{7}$.

Simplify

13. $\frac{2}{5}$ of $\frac{9}{10}$ of 25.

14. $\frac{7}{11}$ of $2\frac{1}{2}$ of $\frac{3}{7}$ of $10\frac{1}{2} - \frac{7}{44}$.

15. $\dfrac{2 + \frac{1}{6} \text{ of } 5\frac{1}{7}}{\frac{5}{7} \text{ of } \frac{1}{2}}$.

16. $(3\frac{1}{2} + 2\frac{1}{3} + 5\frac{1}{4}) \times 2\frac{2}{3}$ of $1\frac{10}{11} - \frac{14}{33}$.

17. $\dfrac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{3} + \frac{1}{4}}$ of $\dfrac{\frac{1}{4} - \frac{1}{5}}{\frac{1}{5} + \frac{1}{6}}$ of $\dfrac{\frac{1}{6} - \frac{1}{7}}{\frac{1}{7} + \frac{1}{8}}$ of 1155.

18. $\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{7}{8}}$.

19. $5\frac{1}{3}$ of $\dfrac{1}{1\frac{1}{3} + \dfrac{1}{2\frac{1}{4}}} \div \dfrac{4\frac{1}{6} + 5\frac{3}{4}}{4\frac{1}{4} + 3\frac{2}{3}}$.

20. $1\frac{2}{5}$ of $\dfrac{1}{1\frac{1}{2} + \dfrac{1}{3\frac{1}{2}}} \div \dfrac{5\frac{1}{3} + 3\frac{1}{5}}{6\frac{1}{5} + 10\frac{1}{3}}$.

21. (i) $\frac{3}{8} - \frac{1}{3} + \frac{7}{24} - \frac{5}{16} + \frac{1}{6}$;

(ii) $\frac{9}{11}$ of $\dfrac{\frac{41}{162} - \frac{9}{49} - \frac{3}{54}}{\frac{4}{9} + \frac{1}{2} - \frac{13}{14}}$ of $(4\frac{2}{5} - 2\frac{1}{2} + \frac{3}{10})$.

Find the value of

22. $\dfrac{5\frac{1}{7} - 3\frac{5}{14}}{2\frac{1}{2} + 3\frac{3}{4}} \times 79\frac{1}{2}$.

23. $\dfrac{2\frac{5}{9} \text{ of } 1\frac{17}{46} - 3}{4\frac{2}{11} - 2\frac{3}{4}} \div 2\frac{2}{11}$.

24. $(2\frac{1}{4}$ of $5\frac{3}{7}) + (3\frac{1}{4}$ of $9\frac{1}{3}) - 10\frac{18}{35}$.

25. $\dfrac{17\frac{6}{7}}{9\frac{1}{11}} + 2\frac{3}{7} \times (9\frac{1}{2} - 2\frac{11}{17}) - \dfrac{5}{1 - \frac{3}{31}}$.

26. $\frac{1}{35}$ of $4\frac{1}{3}$ of $\dfrac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{4} + \frac{1}{5} + \frac{1}{6}}$.

27. How many times is the difference between

$$\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2}} \text{ and } \dfrac{\frac{2}{5} \text{ of } \frac{5}{8}}{\frac{2}{9} \text{ of } 4\frac{1}{2}}$$

contained in their sum?

28. By how much is $\frac{1}{10} - \frac{1}{7} + \frac{3}{5} - \frac{1}{15}$ greater than $\frac{57}{240}$?

Add together

29. $\frac{1}{2}, \frac{1}{6}, \frac{1}{7}, \frac{1}{14}$ and $\frac{9}{56}$.

30. $\frac{4}{13}, \frac{7}{15}, \frac{7}{39}$ and $\frac{3}{65}$.

31. $\frac{2}{3}, \frac{3}{4}, \frac{3}{8}, \frac{7}{12}$ and $\frac{19}{36}$.

32. $\frac{1}{51} + \frac{2}{45} + \frac{3}{38} + \frac{4}{85} + \frac{5}{57}$.

33. Of a regiment of soldiers, $\frac{1}{30}$th are killed or disabled in the first battle, $\frac{7}{29}$ths of the remainder in the second battle, and $\frac{3}{11}$ths of the remainder in the third; if 512 men are left, how many were there at first?

MISCELLANEOUS EXERCISES.

1. Find the sum of $3\frac{11}{16}$, $2\frac{17}{48}$, $1\frac{37}{80}$.

2. Divide $\frac{3}{14}$ of $5\frac{5}{8}$ by $7\frac{17}{49}$.

3. Multiply together $4\frac{3}{14}$ of $6\frac{2}{9}$ of $\frac{25}{33}$ and $5\frac{1}{4}$ of $\frac{22}{35}$.

4. Subtract $1\frac{1}{9}$ of $3\frac{4}{15}$ from $\dfrac{20\frac{5}{11}}{3\frac{1}{33}}$.

5. Subtract $\dfrac{13\frac{4}{7}}{1\frac{37}{42}}$ from $3\frac{4}{7}$ of $19\frac{14}{15}$.

6. Divide $3\frac{1}{2}$ of $1\frac{5}{12}$ by $5\frac{1}{8}$ and the result by $1\frac{2}{7}$.

Reduce to the simplest forms ·

7. $\dfrac{(3\frac{1}{6})\times\dfrac{8}{57}-\dfrac{6\frac{1}{6}-5\frac{4}{5}}{11\frac{1}{3}+\frac{1}{5}}}{(2\frac{1}{9})\div(2\frac{9}{11})}$.

8. $\dfrac{\dfrac{16\frac{2}{3}}{7}+\dfrac{19}{5\frac{1}{4}}-\dfrac{10\frac{8}{9}}{4\frac{5}{11}}\text{ of }1\frac{7}{11}}{\left(\dfrac{2}{3}+\dfrac{6}{7}\right)\left(\dfrac{4}{5}-\dfrac{5}{8}\right)}$.

9. $(\frac{2}{3}\text{ of }\frac{7}{11}-\frac{1}{2}\text{ of }\frac{5}{6})\times\frac{3}{7}\text{ of }\frac{5}{8}$.

10. $\dfrac{13\frac{2}{3}}{11\frac{4}{11}}\text{ of }1\frac{49}{451}-\dfrac{6\frac{1}{2}-5\frac{1}{4}\text{ of }\frac{6}{7}}{4\frac{1}{3}\text{ of }\frac{7}{13}+1\frac{1}{66}}$.

11. $\dfrac{5+2\frac{7}{9}\text{ of }\frac{27}{50}}{30-6\frac{6}{11}\text{ of }2\frac{1}{5}}+\dfrac{6\frac{5}{6}}{23\frac{3}{7}}\text{ of }(6\frac{1}{21}-5\frac{9}{14})$.

12. $\dfrac{44\frac{2}{3}-3\frac{3}{11}\text{ of }10\frac{1}{12}}{3\frac{1}{2}+\frac{7}{65}\text{ of }13\frac{13}{14}}+\dfrac{(5\frac{3}{4}-3\frac{2}{3})\text{ of }2\frac{5}{9}}{1\frac{1}{4}\text{ of }(2\frac{1}{2}+1\frac{1}{3})}$.

13. $\dfrac{100\frac{1}{13}-91\frac{89}{91}}{27\frac{2}{11}+2\frac{117}{143}}-\dfrac{3\frac{3}{7}+6\frac{3}{5}\text{ of }1\frac{11}{39}}{1\frac{7}{11}\text{ of }25\frac{2}{3}+48\frac{1}{6}}$.

14. $\dfrac{3\frac{3}{7}}{\dfrac{4}{5}\left(\dfrac{3}{3\frac{1}{2}}\text{ of }7\right)}+\dfrac{5\frac{5}{8}}{4\frac{1}{6}}\text{ of }\dfrac{4-(3\frac{1}{3}-1\frac{1}{6})}{3\frac{4}{15}-2\frac{1}{60}+\frac{11}{20}}$.

15. $\dfrac{7\frac{2}{3}}{4\frac{2}{11}}\text{ of }\frac{24}{77}-\dfrac{(3\frac{2}{7}-2\frac{5}{6})\text{ of }3\frac{3}{13}}{3\frac{2}{7}-2\frac{5}{6}\text{ of }\frac{24}{119}}$.

16. $\dfrac{2\frac{1}{9}}{5\frac{3}{7}}\text{ of }1\frac{1}{8}-\dfrac{15\frac{3}{4}-4\frac{3}{7}\text{ of }3\frac{1}{2}}{3\frac{5}{9}\text{ of }\frac{3}{8}+2\frac{2}{3}}$.

17. $(\frac{3}{4}\text{ of }\frac{12}{11}-\frac{1}{2})\times\dfrac{1+(\frac{2}{3}\text{ of }\frac{39}{40})\div\frac{78}{84}}{2+\frac{1}{2}+\frac{1}{4}-\frac{13}{9}\div\frac{11}{18}}$.

Reduce to the simplest forms:

18. $\frac{1}{39}$ of $4\frac{1}{3}$ of $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}+\frac{1}{3}$.

19. $\dfrac{\frac{2}{3}+\frac{3}{5}}{\frac{2}{3}-\frac{3}{5}} \times \dfrac{\frac{2}{3}-\frac{2}{5}}{\frac{2}{3}+\frac{2}{5}}$.

20. $\dfrac{\frac{1}{5}-\frac{1}{6}+\frac{1}{9}-\frac{1}{18}}{\frac{1}{4}-\frac{1}{5}+\frac{1}{9}-\frac{2}{45}}$.

21. $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$ of $\dfrac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}}$ of $\dfrac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{6}+\frac{1}{7}}$ of 585.

22. What number is that from which if $\frac{7}{15}-\frac{3}{12}$ be deducted and to the remainder $\dfrac{\frac{4}{9}}{1\frac{1}{4}}$ be added, the sum will be $3\frac{23}{36}$?

23. Simplify the expression $\dfrac{3\frac{1}{2}}{7\frac{1}{4}} \times \dfrac{17\frac{2}{5}}{5\frac{8}{11}}$.

24. The price of coals rises from 15s. 9d. to 18s. 6d. per ton. What is the extra expense to a firm laying in 24 tons, an amount which is $\frac{1}{5}$ less than their usual supply?

25. Add together $2\frac{1}{2}$, $3\frac{5}{8}$, $\frac{4}{9}$, and $\frac{13}{72}$.

26. (i) From $8\frac{3}{7}$ take $6\frac{4}{5}$; (ii) multiply $4\frac{1}{5}$ by $\frac{10}{63}$.

27. Simplify $(6\frac{3}{7}+1\frac{7}{9}) : (3\frac{1}{3}-2\frac{1}{5})$.

28. The value of a ship and its cargo is £17,600, if the value of the cargo is seven times that of the ship, find the value of the cargo, and the ship.

29. Two taps can separately fill a cistern in 9 and 12 minutes, and the waste pipe can empty it in 8 minutes. How long will the two taps together take to fill the cistern when the waste pipe is open?

30. A man pays a tenth of his income in rates and taxes, and a twelfth in insurances. He has left £492 13s. 1d. What is his income?

31. A sovereign consists of 22 parts by weight of pure gold to 2 parts alloy, and it weighs 123·274 grains troy. Neglecting the value of the alloy, what is the value of pure gold per ounce troy to the nearest penny?

32. A can reap $\frac{4}{9}$ of a field in $2\frac{2}{3}$ days. B can reap $\frac{3}{7}$ of the field in $4\frac{1}{2}$ days. In what time can A and B reap it when working together?

33. A cistern can be emptied by one tap in 4 hours and by another tap in 6 hours. In what time will the cistern be emptied when both taps are opened?

34. Three pipes are used to fill a tank. It is found that the first tap will fill the tank in 3 hours, the second in 5 hours, and the third in 6 hours. What time will be required when all three are used together?

35. If $\frac{4}{5}$ of a plot of land is worth £36, what is the value of the whole plot?

36. Reduce £3. 14s. 8d. to the fraction of £7.

37. If $7\frac{2}{3}$ yards of flannel cost £1. 6s. 10d., what is the price per yard?

38. Simplify (i) $\frac{2}{3}+\frac{4}{5}+\frac{6}{7}-\frac{34}{105}$.
(ii) $(\frac{10}{11}-\frac{8}{9})+(6\frac{1}{2}\div 1\frac{1}{12})$.

39. Find the sum of $\frac{1}{7}$ of 16s. 11d. and $\frac{13}{200}$ of 16s. 8d.

40. Simplify $(\frac{3}{4}+\frac{1}{3}-\frac{1}{8})\times\frac{6}{25}\div\frac{11}{16}$.

41. Add together $\frac{2}{3}$ of £1, $\frac{5}{8}$ of 18s. 6d., and $\frac{3}{11}$ of 6s. 5d.

42. A tank when $\frac{1}{4}$ full of oil weighs 3 lbs. 15 oz., and when $\frac{3}{5}$ full weighs 7 lbs. Find the weight of the tank (*a*) when empty, (*b*) when full of oil. [L.C.U.]

43. A piece of land consists of $2\frac{1}{2}$ acres, $\frac{1}{10}$ of it is occupied by a house, $\frac{1}{4}$ of the remainder as a kitchen garden, and the rest as a paddock. Find the area of the paddock. [N.U.T.]

44. A roll of calico is 32 yds. long; 20 yds. are cut off. What fraction of the roll remains? If the remainder is worth 7s. 3d., what was the value of the roll? [N.U.T.]

45. A penny weighs $\frac{1}{3}$ oz. and consists of an alloy of 95 parts copper, 4 parts tin, and 1 zinc. If copper is worth £66 per ton, tin £170 and zinc £25, find the value of the metal in 240 pence. [L.C.U.]

46. The difference between a quarter of x and a sixth of x is 1·7 inches. Find x in feet. [N.U.T.]

Summary.

A **Fraction** is a part or parts of a whole.

To add or subtract Fractions.—Reduce, if necessary, to a common denominator, then add or subtract as required and place the sum or difference over a common denominator.

To multiply Fractions.—Multiply the numerators together for a new numerator and the denominators for a new denominator.

To divide Fractions.—Reduce, if necessary, both fractions to a simple form, invert the divisor, and proceed as in multiplication.

To reduce a Vulgar Fraction to its lowest terms.—Divide both numerator and denominator by any factors common to both, or find the G.C.M. of numerator and denominator, and divide both by it.

CHAPTER III.

DECIMAL FRACTIONS.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF DECIMALS.

A DECIMAL FRACTION is a fraction in which the denominator is some power of 10, such as 10, 100, 1000, etc.

Decimal Fractions. In a number such as 5555 (five thousand five hundred and fifty-five) it is seen that the same figure is used throughout, but the values of the different fives vary considerably. For every place a 5 is removed to the left its value is increased ten times. Or, in reading a set of figures from left to right, each figure has one-tenth the value it would have if it were moved one place to the left.

Decimal fractions are obtained by using the same notation to indicate numbers less than unity, each digit having one-tenth of the value that it would have if it stood one place farther to the left. Thus 5·555 means $5+\frac{5}{10}+\frac{5}{100}+\frac{5}{1000}$. The point used in this notation is called the decimal point, or simply the point. The above number would be read as "five, point, five, five, five." In a similar manner, 8·073 would be read as *eight, point, nought, seven, three.*

By means of the foregoing explanation the following table, giving in a few cases the relative values of the digits to the left and right of the decimal point, will be readily followed:

Thousands	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths	Ten Thousandths
4	3	2	1	·	2	3	4	5

Addition and Subtraction of Decimal Fractions.—When decimal fractions are used, the simple rules of Arithmetic can be applied readily and easily. The addition and subtraction of decimal fractions are performed exactly as in the case of whole numbers; the only precaution necessary to prevent mistakes is to keep the decimal points under each other. The following examples will show what is meant.

Ex. 1. Add together 36·053, ·0079, ·00095, 417·0, 85·5803, and ·00005.

```
 36·053
   ·0079
   ·00095
417·0
 85·5803
   ·00005
538·64220
```

Ex 2. Subtract 578·9345 from 702·387.

```
702·387
578·9345
123·4525
```

In each case the decimal points are placed under each other, and the addition and subtraction are carried out as in the case of whole numbers.

EXERCISES. VI.

Add together

1. ·00946, 8·0203, 156·98, and 45·9876.
2. 131·121, 2·86754, ·00102, and 63·2059.
3. 26·489, 1·2687, 342·76, and ·00923.
4. 352·789, 1·0021, 11·4218, and ·0102.
5. ·000495, 16·96, 5·043, and 60·001607.
6. 226·918706, 21·0674, 3·09631, and ·25732.
7. 47·01913, 635·77, ·00187, and 352·9.
8. ·6183, 151·65, 9·00074, and 58·0961.
9. 362·134, ·031427, 3·076, and 4·987.
10. ·0487, 151·65, 9·00074, and 64·4683.
11. Add to the sum of 13·0009, 4·5672, 1·89, and ·007999 quantity which shall make a total of 20.
12. Add together ·7055, 324·88, 7·08213, and ·0621.

Subtract

13. ·07063 from 1·003032.
14. 21342·071 from 30146·03527.
15. 28·3097625 from 41·09317.
16. 28·7643 from 37·593165.
17. 30·85762 from 75·4017.
18. 53·58107 from 61·62005.
19. 49·8703 from 85·02307.

Add together

20. 341·0764, 2·987643, ·0047 and ·4698732.

21. 47·003, 99, 910009, ·999 and ·00054321.

22. 2·0013, 117·9983, 47·231801 and ·04713283.

23. 504·603, ·047625, 37·0078 and 14·80607.

24. 43·587, ·60935, 501·097 and 1·8329.

Subtract

25. 357·908 from 5032·07.

26. 129·6398 from 431·265.

27. 15·01853 from 47·06.

28. 48·098675 from 121·0768.

29. Add together 14·9256, ·00946, 8·0203 and 125·687.

30. Subtract 436·937 from 501·0258.

31. Add together 4·0255, ·60843, 4307·0582 and 327·8645.

32. Add together 3·40765, 537·063 and ·84379. Subtract the result from 601·0173

33. Add together 4·00607, ·617634 and 376·473, and subtract the result from 400.

Multiplication of Decimals.—The process of multiplication is carried out in the same manner as in the case of whole numbers Commencing from the right, as many digits are pointed off in the product as there are digits following the decimal points in the multiplier and the multiplicand added together.

Ex. 1. 36·42 × 4·7.

Multiplying 3642 by 47, we obtain the product 171174. As there are two digits following the decimal point in the multiplicand and one digit following the decimal point in the multiplier, we point off three digits from the right of the product, giving as a result 171·174.

Ex. 2. ·000025 × ·005.

Here 25 × 5 = 125.

In the multiplicand there are 6 digits following the decimal point, and in the multiplier 3. Hence the product is ·000000125. The positions to the right of the decimal point, occupied by the six digits and the three digits referred to, are often spoken of as "decimal places"; thus ·000025 would be said to consist of six decimal places, or two significant figures.

A similar method is used when three or more quantities have to be multiplied together.

Ex. 3. $2{\cdot}75 \times {\cdot}275 \times 27{\cdot}5$.

The continued product of $275 \times 275 \times 275$ is found to be 20796875. Now there are two decimal places in the first multiplier, three in the second, and one in the last. This gives a total of 6 decimal places to be marked off. Hence the required product is 20·796875.

In addition to applying this rule for determining the number of decimal places, the student should mentally verify the work wherever possible. Thus, in the above example, by inspection, it is seen that ·275 is nearly $\frac{1}{4}$, and $\frac{1}{4}$ of 27 is 9. This result multiplied by 2 shows that the final product will contain two significant figures, followed by decimal places. Rough checks of this kind often prevent absurd mistakes finding their way into the work. Moreover, they lead the student to rely upon his own worked answers.

Ex. 4. $73{\cdot}0214 \times {\cdot}05031$.

The product obtained as in previous cases is 3·673706634.

In practice, instead of using the nine decimal places in such an answer as this, an approximate result is, as a rule, more valuable than the accurate one. The approximation consists in leaving out, or, as it is called, *rejecting* decimals, and the result is then said to be true to one, two, three, or more decimal places, depending upon the number of decimal places which are retained in the result. The rule adopted when decimals are rejected is as follows: If any rejected figure is 5, or greater than 5, add one to the preceding figure; but if the rejected figure is less than 5, the preceding figure remains unaltered.

Thus, in the last example the result true to one decimal place is 3·7; the rejected figure 7 being greater than 5, the preceding figure 6 is increased by unity. The result true to two places is 3·67; the rejected figure 3 is less than 5, and the preceding figure is therefore unaltered. In a similar manner the result true to three and four decimal places would be 3·674 and 3·6737 respectively.

When, as in Ex. 2, the result is a decimal fraction, the decimal point being followed by a number of zeros, the number of decimals must obviously be sufficient to include one or more significant figures. Thus, in the example referred to, the three significant figures 125 must be retained.

EXERCISES. VII.

1. Show that the number 32·1065 will be multiplied by 100 if the decimal point be moved two places to the right.

Multiply

2. 8·03 by ·008.
3. ·0016 by ·008.
4. 1·005 by ·005.
5. 183·026 by 2·000999.
6. ·7281075 by ·096008.
7. 14·95 by ·00734.
8. 16·02 by ·0007.
9. ·002897 by 3020.
10. ·47923 by 90·24.
11. 423·209 by ·00785.
12. 47·308 by 17·0009.
13. Multiply 4·62 by ·025 and the product by ·0019.

Multiply

14. 13·064973 by 26·035.
15. 150·079 by 14·00014.
16. 4632700 by ·0172.
17. 7·51 by ·0016.
18. ·0625 by 1·74.
19. ·049 by 3·417.
20. 16·02 by ·0007.
21. 32·47 by ·0033.
22. 2·574 by ·00005.
23. 12·345 by ·00014.
24. Multiply ·671 by ·42 and by ·0420.

Division of Decimals. The division of one quantity by another when decimals enter into the operation, is performed exactly as in the case of whole numbers. The process can be best explained by an example as follows:

Ex. 1. Divide ·7 by ·176.

This may be described as finding a number, which, when multiplied by ·176, gives the product equal to ·7.

Though decimals may be divided as in the case of whole numbers, care is necessary in marking off the decimal point. In the present, and in all simple cases, the position of the decimal point is evident on inspection. Practically, it is often convenient to multiply both terms by 10, or some multiple of 10—100, etc.—and so obtain at once without error the position of the unit's figure, and hence of the decimal point.

```
1·76)7·00(3·97
     5 28
     1 720
     1 584
       1360
       1232
        1280
```

Thus in the above example, multiplying both terms by 10, we have to divide 7 by 1·76, and it is at once seen that the number required lies between 3 and 4. This determines the position of the unit's figure. As 7·0 is unaltered by adding any number of ciphers to the right, we add two for the purpose of the division: multiplying 1·76 by

3 we obtain 5·28, which, subtracted from 7·00, gives a remainder 1·72; to this we affix a cipher and carry on the division as far as necessary; when this is done, we find $\cdot 7 \div \cdot 176 = 3\cdot 9772727$.

It will be seen that the ordinary method of performing division necessarily requires considerable space, especially when there are several terms in the quotient.

Italian Method.—Another method, often referred to as the *Italian* method, in which only the results of the several subtractions are written down, is often used, the method of procedure is as follows: Note, as before, that 1·76 will divide into 7; then since $3 \times 6 = 18$, the 8 is not written down but is instead mentally subtracted from 10, leaving 2. Next $3 \times 7 = 21$ and 1 carried makes 22; the 2 is again not written down, but instead, after the addition of unity from the last process, we say 3 from 10 = 7. In a similar manner the remaining figure is obtained; the next row of figures is arrived at in like manner and so on. Comparing the two examples it will be seen that as at each step of the work one line of figures is dispensed with, the working takes up far less room than is the case in the ordinary method.

```
1·76 ) 7·00 ( 3·97
       1 720
         1360
          1280
```

Recurring Decimals.—The above example illustrates another important point, viz., that although the quotient may in some cases consist of many digits following the decimal point, it may come to an end eventually; but in other cases, the quotient goes on without end, and is known as a *recurring* decimal. It is not often necessary in practical calculations to continue any operation in decimals to more than the third or fourth place, and in the majority of cases, if the quotient is correct to two significant figures it is amply sufficient. In the previous example, the quotient correct to the second decimal place would be 3·98.

It is obviously bad in principle to use more figures than are essential for the work in hand; for they are not only unnecessary, but give additional trouble, and also increase the risk of making mistakes. In many cases, students are found to work with ten or more decimal figures, when, owing to errors of observation, or measurement, or to slightly incorrect data, even the first decimal place may not be trustworthy. It is, of course, inadvisable to add an error of arithmetic to an uncertainty of

measurement or data, but even a slight error is preferable to working out ten or fifteen places of decimals to a practical question when two or three decimal places at most are sufficient.

EXERCISES. VIII.

Divide

1. ·0468 by 29·25. **2.** 7·777 by 35·35.

3. ·1154255 by ·00115. **4.** 26·751 by ·000925.

5. 5·4 by ·00072 and the quotient by 1470568.

6. 3·425 by ·002192. **7.** 157·875 by 36·8.

8. 362·97 by ·00545. **9.** 8·125 by 2·175.

10. ·005 by ·425. **11.** ·25 by ·00325.

12. ·002346 by ·001825. **13.** 9·0225 by 22·5.

14. 22·5 by 9·0225. **15.** 90·915 by 209.

16. ·0006594 by ·0021. **17.** 13·5 by ·001125 and verify the result.

18. ·085712 by 25·603. **19.** 712·8576 by 28560.

20. 360·221286 by 898·98. **21.** 51·2705 by ·0205.

22. 3565·345 by 36·05. **23.** 14·259 by 582.

24. 21·06 by 64·8. **25.** 79682·44 by ·0172.

26. 18596·508 by 98760. **27.** 3681·15133 by 90·0037.

28. 1·047034 by ·0302 **29.** 178·6686 by ·0642.

30. ·0141009 by 47·003 **31.** 1027·5 by ·875.

32. ·0325 by 6·5.

33. From a rod a yard long, portions each ·057 of an inch long are cut off. How many such portions can be cut off, and what will be the length of the remaining piece?

34. Divide the product of ·035 and ·0056 by ·00007.

35. How many times can ·013 be subtracted from 125·78, and what will be the magnitude of the remainder?

36. The length of a strip of plate is 14·578 inches. Neglecting the loss in cutting, how many pieces ·053 inches long can be cut from it; and what will be the length of the remaining piece?

MISCELLANEOUS EXERCISES. IX.

1. Add together 18·3 and 21·065. Subtract 17·099 from 25·1 and divide the former by the latter.

2. Divide the product of 851 and ·00016 by ·032.

3. Find the value of $(65 - 39{\cdot}61) \times 39{\cdot}61$.

4. Find the product of 1·005 and ·097 and the quotient of ·0036 divided by ·0144.

5. Find the product of ·0256 and 1·0071 and divide the product by 2·7975.

6. Multiply 91·77 by ·091 and divide the product by 2·73.

7. Multiply ·02019 by 52·03, and divide ·0431 by ·0044.

8. Divide (i) ·0175 by 17·5, and (ii) 17·5 by 7·0175.

9. Multiply and divide 4203·18 by 2·3175.

10. Find the value of (2·37 × ·093) ÷ ·0005.

Divide

11. ·75 by 8 and 2·5 by 3·2. 12. 1 by ·013 and ·00042 by ·007.

13. 2·1170 by ·0073 and ·0257 by ·0041.

14. 172·9 by ·142 and 17·415 by ·387.

15. Divide ·736 by 2·85 and 2·85 by ·7036. Find the product formed by multiplying the two quotients together.

16. Find (i) the product of 1·005 multiplied by 0·097, and (ii) the quotient of ·0036 divided by ·0144.

17. Find the value of

$$(0{\cdot}1 \times 0{\cdot}01 \div {\cdot}0002) - ({\cdot}6375 \times \tfrac{1}{17} \div {\cdot}125).$$

18. Divide $0{\cdot}2 \times 0{\cdot}3 \div {\cdot}00012$ by $(5{\cdot}719 \times \tfrac{1}{19}) + (0{\cdot}3 \times 1{\cdot}2 \times 9)$.

19. Reduce to simplest form the following.

$$({\cdot}002 \times 36{\cdot}25 \div {\cdot}029) - (102{\cdot}85 \times {\cdot}04 \div 1{\cdot}7).$$

20. $({\cdot}0011 \times {\cdot}091 \div {\cdot}0035) + ({\cdot}015 \times {\cdot}507 \div {\cdot}39)$

21. $({\cdot}0057 \times 2{\cdot}09 \div {\cdot}361) - ({\cdot}00165 \times {\cdot}077 \div {\cdot}0105)$

22. Find the product of ·0256 and 1·0071, and divide the product by 2·7975.

23. Reduce to its simplest form

$$\frac{1{\cdot}1931 \div 5{\cdot}82}{{\cdot}225 + {\cdot}2666}.$$

Summary.

Decimals.—Addition and subtraction are performed as in simple addition and subtraction; the figures are set down so that the decimal points are one above the other.

Multiplication of Decimals.—Proceed as in the case of the multiplication of whole numbers, and when the product is obtained, point off from the digit on the extreme right as many decimal places as there are in the multiplier and multiplicand together.

Division of Decimals.—Make the divisor a whole number by moving the decimal point as many places to the right in both divisor and dividend as may be necessary. Proceed as in the division of whole numbers, inserting the decimal point in the quotient as soon as in the course of the work any figure is used from the decimal part of the dividend.

CHAPTER IV.

CONTRACTED MULTIPLICATION AND DIVISION. CONVERSION OF VULGAR TO DECIMAL FRACTIONS. RECURRING DECIMALS. DECIMALS OF CONCRETE QUANTITIES.

Contracted Multiplication and Division.—The results of all measurements are at the best only an approximation to a true result. The accuracy of expression is, it is true, increased by extending the number of decimal figures in the result, but it should be carefully noted that the accuracy of any result does not depend on the number of decimal places to which the result is calculated, but it does depend on the accuracy with which the measurements or observations are made.

In any result obtained the last decimal place may not be accurate, but the figure preceding should be as accurate as possible. It is therefore advisable for the sake of accuracy to carry the result to one place more than is required in the result.

It is at once evident that loss of time will be experienced if we multiply together two numbers in each of which several decimal figures occur, and after the product is obtained reject several decimals. Especially is this the case in practical questions in which the result is only required to be accurate to the first or second place of decimals. In all such cases what is known as Contracted Multiplication may be used.

Contracted Multiplication.—In this method the multiplication by the *highest* figure of the multiplier is first performed. By this means the first partial product obtained is the most important one.

The method can be shown and best understood by an example.

Ex. 1. Multiply ·006914 by 8·652.

The product of the two numbers can of course be found by the ordinary methods; and to compare the two methods, "ordinary" and "contracted," the product is obtained by both processes:

*Ordinary Method.**	*Contracted Method.*
6914	6914
8652	2568
13828	55312
34570	4148~~4~~
41484	346~~70~~
55312	14~~828~~
·059819928	·059820

The ordinary method will be easily made out. In the contracted method it will be seen that **the figures in the multiplier are reversed,** and the process continued as follows: Multiply first by 8, so obtaining 55312; next by 6—this step we will follow in detail—$6 \times 4 = 24$, the 4 is not written down (but if written down it is cancelled as indicated), and the 2 is carried on. Continuing, $6 \times 1 = 6$, and adding on 2 gives 8. Next, $6 \times 9 = 54$, the 4 is entered; and 6×6 is 36, this with the 5 from the preceding figure gives 41, hence the four figures are 4148.

In the next line, multiplying by 5, we can obtain the two figures 0 and 7, but as these are not required unless there is some number to be carried, it is only necessary to obtain 69×5, and write down the product 345, add 1 for the figure rejected, making 346; finally, as 2×9 will give 18, and therefore we have to carry 1, we obtain $2 \times 6 = 12$, together with the one carried from the preceding figure gives 13, add 1 for the figure (8) rejected, which gives 14. Adding all these partial products together we obtain the product required.

Thus, in the second row one figure is rejected, in the next row two figures, and in the last row three figures are left out.

It must be noticed that when the rejected figure is 5 or greater, the preceding figure is increased by 1, also that the last figure of the product is not trustworthy. Having noted (or cancelled) the rejected figures, as will be seen from the example, the decimal point is inserted as in the ordinary method, *i.e.* marking off in the product as many decimal places as there are in the multiplier and multiplicand together.

Though the multiplier is very often reversed, this is not neces-

sary, except to avoid mistakes. The multiplier may be written in the usual way, and the work will then proceed from the left-hand figure of the multiplier, *i.e.* the work is commenced by multiplying by 8 and not by 2.

EXERCISES. X.

Multiply

1. 250·635 by 8·056.
2. 89·8998 by 3·047.
3. 240·35 by 7·0923.
4. Multiply together ·543, 643·2 and ·3008.

Multiply

5. 65·0843 by ·0925
6. 82·009 by 3·685
7. Multiply together 4·073, ·0032 and ·09.

Multiply

8. 16·5634 by 1·02.
9. 42·359 by ·623.
10. 58·301 by ·758
11. 17·4987 by ·37004.
12. 168·05 by 1·0069.
13. ·0008679 by 496·038.
14. 589·0067 by 3·1008.
15. 4·86923 by ·007655.
16. 8·0198 by ·001077.
17. 41·0027 by 31·093.
18. 500·173 by ·00376.
19. 471·013775 by 420·17.
20. 709·285 by 1·0069.
21. 8561·02 by 5·6039.
22. ·0057632 by 2·873.
23. 4·3129 by 46000.
24. 17·081 by 3·00091.
25. Multiply together 11, 1·1, ·0011 and 1·25.
26. Multiply together ·275, 2·75 and 27·5

Multiply

27. ·00734 by 7·164.
28. 8·07639 by 28·73.
29. 73·0214 by 50·31.
30. 36·2894 by 8·93.
31. 11·416 by 3·16.
32. 512·1875 by 26·8.
33. 28·395 by ·00114.

Contracted Division.—It is assumed that the student is familiar with the ordinary method of obtaining the quotient in the case of division, but the long process of division can also be advantageously contracted. The method of doing this will be clear from the following worked example.

Ex. 1. Divide ·03168 by 4·208.

We shall work this example by the contracted method alone.

To begin with, the number 7 is obtained by the usual process of division. By multiplying the divisor by 7 the product 29456

is arrived at. When this is subtracted from 31680 the remainder 2224 is left. It is seen that if we drop or cancel the 8 from the divisor 4208, thus obtaining 420, it can be divided into the remainder 2224, five times. In multiplying by 5 we take account of the 8, thus, as 5×8 is 40, we do not enter the 0 but carry on the 4; but $0 \times 5 = 0$, and adding 4, we see this is the figure to be entered. Now proceed to the next and the following figures, obtaining in the usual way 2104, subtract this from 2224, and the remainder 120 is obtained. Proceeding in like manner with the multiplier 2, we obtain 84, which, subtracted from 120, leaves 36, and our last figure in the quotient is 9. By the method described on p. 26 the answer is ·007529.

```
4208 ) 31680 ( 7529
       29456
        2224
        2104
         120
          84
          36
          36
```

The above example shows that the method of contracted division consists in leaving out or, as it is called, rejecting a figure at each operation. Any number which would be added on to the next figure by the multiplication of the rejected figure is carried forward in the usual way. To avoid mistakes it may be convenient either to draw a line through each rejected figure of the divisor, or to place a dot under it.

EXERCISES. XI.

Divide

1. 2515·611 by ·0785.
2. 17·228 by ·584.
3. 6·9591555 by 843·534.
4. 99·721 by ·834.
5. 64·375 by 9·573.
6. 2·873 by 48·607.
7. 11·7813 by ·2724.
8. ·304775 by 59·678.
9. 3·743 by 52·804.
10. 174·13238 by 5·615.
11. 2·821005 by 29·332.
12. 110246 by 26·063.
13. 184·13 by 5·615.
14. 155·7 by 358.
15. 471·6 by 1·235.
16. 1·5104 by 40·13.
17. 246·87 by ·7777.
18. 1·11 by 8·0908.
19. 42·266 by ·05838.
20. 311 010 by ·076359.
21. 1·132041 by 24·0604.
22. 129·8 by ·234.
23. ·1115 by 45·9.
24. 108·78 by 444.
25. 147·5 by 235.
26. ·013119 by 2·403.
27. ·39908 by 248·03.
28. 9·22 by ·00375.
29. ·0222946 by 6·4065.
30. 4·0975 by ·2980.
31. ·169 by ·0013.
32. 49·7 by ·0025.

Divide.

33. ·00003 by ·001. **34.** 4·65131 by 596·8.

35. 183·026 by 2·03.

36. The sum of 1·2854 and ·763 by their difference.

37. ·049855 by 1·3569.

Conversion of a Decimal to a Vulgar Fraction.—Having already found (p. 22) that ·3 is merely a convenient way of writing $\frac{3}{10}$, and ·03 of writing $\frac{3}{100}$, it will be obvious that in the two cases just referred to $\frac{3}{10}$ and $\frac{3}{100}$ are the vulgar fractions required. In a similar manner ·2 = $\frac{2}{10}$, and ·02 = $\frac{2}{100}$. In each case the numerator and denominator are divisible by 2, hence we write ·2 = $\frac{2}{10} = \frac{1}{5}$, and ·02 = $\frac{1}{50}$.

From a consideration of such examples we may state the rule for the conversion of a decimal into its equivalent vulgar fraction as follows:

Take the given decimal fraction for the numerator and for a denominator write down 1 followed by as many zeros as there are decimal places in the given decimal fraction; if necessary reduce the fraction so obtained to its lowest terms.

Some Common Values.—There are many decimal fractions of such frequent occurrence in practice that it may be advisable to commit them and their equivalent vulgar fractions to memory.

Thus ·125 = $\frac{125}{1000} = \frac{1}{8}$; ·25 = $\frac{25}{100} = \frac{1}{4}$; ·375 = $\frac{375}{1000} = \frac{3}{8}$; ·5 = $\frac{5}{10} = \frac{1}{2}$; ·75 = $\frac{75}{100} = \frac{3}{4}$.

It will be noticed that by remembering the first of the above results the other fractions can be obtained by multiplying it by 2, 3, etc., or in each case the result is obtained by mentally dividing the numerator by the denominator.

Conversion of a Vulgar to a Decimal Fraction.—To convert a vulgar fraction to a decimal fraction, reduce the vulgar fraction to its lowest terms and then divide its numerator by its denominator.

Ex. 1. $\frac{3}{8} = 3 \div 8 = \cdot375$; $\frac{7}{8} = 7 \div 8 = \cdot875$.

Ex. 2. $\frac{1}{160} = \cdot00625$. *Ex.* 3. $\frac{54}{125} = \cdot432$.

In many cases it will be found simpler and easier to reduce a fraction to its equivalent decimal if the numerator and denominator are first multiplied by some suitable number.

Ex. 4. Reduce $\frac{7}{250}$ to a decimal.

Multiplying by 4 we get $\frac{28}{1000} = \cdot028$.

In a similar manner $\frac{1}{25} = \frac{4}{100} = \cdot04$.

Other examples can be worked in like manner.

In some cases the figures in the quotient do not stop, and we obtain what are called **recurring** (they are also called **repeating**, and sometimes **circulating**) decimals.

Ex. 5. $\frac{1}{3} = \cdot333 \ldots$

The result of the division is shown by as many threes as we care to write. The notation $\cdot\dot{3}$ is used to denote this unending row.

Ex. 6. Again $\frac{2}{3} = \cdot666 \ldots = \cdot\dot{6}$.

In each of these, and in similar cases, the equivalent vulgar fractions are obtained by writing 9 instead of 10 in the denominator, thus $\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$, etc. In a similar manner $\frac{1}{7} = \cdot142857$, and these figures again recur over and over again as the division proceeds, hence $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$.

When necessary to add or subtract recurring decimals, as many of the recurring figures as are necessary for the purpose in hand are written, and the addition or subtraction performed in the usual manner. With a little practice the student soon becomes familiar with the more common recurring decimals.

Any decimal fraction, such as $\cdot\dot{3}$, $\cdot\dot{1}4285\dot{7}$, in which all the figures recur is called a **pure recurring decimal**; the equivalent vulgar fraction is obtained by writing *as a numerator the figures that recur, and for the denominator as many nines as there are figures in the recurring decimal.*

When the decimal point is followed by some figures which do not recur and also by some which do recur, the fraction is called a **mixed recurring decimal**, and the equivalent fraction is obtained by *subtracting the non-recurring figures from all the figures to obtain the numerator, and as many nines as there are recurring figures, followed by as many cyphers as there are non-recurring figures for the denominator.*

***Ex.* 7.** **Express as a vulgar fraction the recurring decimal $\cdot1\dot{2}\dot{3}$.**

Here there are two recurring figures and one not recurring,

$$\therefore \ \cdot1\dot{2}\dot{3} = \frac{123-1}{990} = \frac{122}{990} = \frac{61}{495}.$$

Ex. 8. $\cdot32\dot{6}5\dot{7} = \frac{32657-32}{99900} = \frac{32625}{99900} = \frac{145}{444}.$

EXERCISES. XII.

1. Reduce $\frac{7}{12}$ to a decimal and prove your method correct.

2. Prove that every fraction can be reduced to either a terminating or a circulating decimal. Give an example of each.

3. Reduce to a decimal $\frac{3}{5} + \frac{7}{8} + \frac{9}{10} + \frac{7}{32}$.

Reduce to decimal fractions

4. $\frac{3}{8}$ of $2\frac{1}{2}$; $\frac{7}{250}$. 5. $\frac{5}{6}$ of $\frac{7}{8}$ of $1\frac{1}{2}$.

6. Find the sum of $3\frac{2}{5} + 4\frac{1}{8} + 1\frac{11}{40} + 3\frac{18}{25}$.

Reduce to decimal fractions

7. $\frac{1001}{1701}$. 8. $5\frac{5}{640} + \cdot 75$ of $\frac{6}{5}$ of $7\frac{1}{2}$.

9. $\frac{3}{64} + \frac{255}{256} + \frac{3472}{200}$.

10. Reduce to vulgar fractions ·365, ·125, ·0035, 8·06 and 8·75.

11. Find the vulgar fraction which is equal to the sum of 15·3125 and 12·0075 divided by their difference

12. Find to four decimal places the value of

$$\frac{\frac{1}{2} + \frac{3}{5} + \frac{5}{7}}{\frac{5}{2} + \frac{4}{5} + \frac{1}{7}}.$$

13. Reduce $\frac{2}{3} + \frac{5}{8} + \frac{1}{16} - 1\frac{1}{6}$ to a single fraction, and convert that fraction into a decimal.

14. Reduce to its simplest form

$$\frac{\frac{5}{8} - \frac{2}{5}}{\frac{4}{3} - \frac{1}{7}},$$

and convert the result into a decimal.

15. Reduce $\frac{1}{2} - \frac{3}{5} + \frac{7}{8} - \frac{11}{25} + \frac{117}{125}$, and express the result as a decimal.

16. Explain what is meant by a Recurring Decimal, and find the vulgar fractions equivalent to the decimals

$\cdot 1\dot{2}\dot{3}$, $11\cdot 12\dot{3}\dot{4}$ and $\cdot 0034\dot{5}6\dot{8}$

17. Divide $\cdot 2\dot{7}$ by $75\cdot\dot{7}\dot{5}$, and express the answer as a decimal.

18. Find the sum, difference, product and quotient of $\cdot 96\dot{3}4\dot{5}$ and $\cdot\dot{3}$.

19. Find the value of $\cdot 3076 \times 1\cdot 0\dot{7}\dot{2} \div \cdot 008$.

20. Express as a vulgar fraction $\cdot\dot{5}3\dot{6}$.

21. Express the product of 4·625 and $\cdot\dot{0}2\dot{7}$ as a vulgar fraction in its lowest terms.

22. Multiply $\cdot 3\dot{1}4285\dot{7}$ by $\cdot\dot{6}\dot{3}$. 23. Multiply $\cdot\dot{2}8571\dot{4}$ by $\cdot 1\dot{1}\dot{6}$.

24. Express the difference of $\cdot 2\dot{2}8571\dot{4}$ and $\cdot\dot{4}2857\dot{1}$, also $1\cdot 5\dot{3}8461\dot{5}$ and $\cdot\dot{0}7692\dot{3}$, as vulgar fractions in their lowest terms.

25. Express the quotient of $\cdot\dot{4}\dot{2}$ divided by $\cdot 0\dot{2}\dot{2}$.

Decimals of Concrete Quantities.—It is often necessary to express a given quantity as a fraction of another given quantity of the same kind. Thus, in the case of £1. 15s., it is obvious that 15s. $= \frac{15}{20}$ of 20 shillings, and thus £1. 15s. may be written £$1\frac{3}{4}$; or, as $\frac{3}{4} = \cdot 75$, we may also write £1. 15s. as £1·75.

Ex. 1. To reduce 10d. to the decimal of a pound,

As there are 240 pence in £1,

$$\therefore \text{ required fraction is } \tfrac{10}{240} = \tfrac{1}{24} = £\cdot 041\dot{6}$$

Ex. 2. Express 6 days 8 hours as the decimal of a week.

As there are 24 hours in a day,

$$6 \text{ days } 8 \text{ hours } = 6\tfrac{8}{24} = 6\tfrac{1}{3} \text{ days},$$

$$\therefore 6 \text{ days } 8 \text{ hours} = \frac{6\dot{3}}{7} = \cdot 90476\dot{1} \text{ week.}$$

Ex. 3. Reduce 5d. to the decimal of 1s.

$$\tfrac{5}{12} = \cdot 41\dot{6}.$$

Ex. 4. Express in furlongs and poles the value of ·325 miles.

Here multiplying by 8 the number of furlongs in a mile, we obtain 2·6, and multiplying the remainder ·6 by 40 (the number of poles in a furlong) we get 24 poles.

Hence ·325 mile = 2 fur. 24 po.

$$\begin{array}{r} \cdot 325 \\ 8 \\ \hline 2\cdot 600 \\ 40 \\ \hline 24\cdot 0 \end{array}$$

Given a decimal fraction of a quantity, its value can be obtained by the converse operation to that described.

Ex. 5. Reduce 9 inches to the decimal of a foot.

There are 12 in. in a foot. Hence the question is to reduce $\frac{9}{12}$ to a decimal.

$$\therefore 9 \text{ in.} = \cdot 75 \text{ ft.}$$

Ex. 6. Find the value of ·329 of £1.

The process is as follows: First multiplying by 20 we obtain the product 6580, and marking off three decimals we get the value 6·580 shillings. In a similar manner multiplying by 12 and 4 as shown, we obtain the value of ·329 of £1, which is read as 6 shillings 6 pence 3 farthings and ·84 of a farthing.

The result could be obtained also by multiplying ·329 by 240, the number of pence in £1, giving 78·96d. and afterwards reducing to shillings, etc.

$$\begin{array}{r} \cdot 329 \\ 20 \\ \hline 6\cdot 580 \\ 12 \\ \hline 6\cdot 960 \\ 4 \\ \hline 3\cdot 840 \end{array}$$

Ex. 7. Find the number of feet and inches in ·75 yard.

Here $\quad \cdot 75 \times 3 = 2 \cdot 25$ ft.,

and $\quad \cdot 25$ ft. $= \cdot 25 \times 12$ in.

$\quad = 3$ in.

$\therefore \quad \cdot 75$ yard $= 2$ ft. 3 in.

EXERCISES XIII.

1. Add together five sevenths, three-sixteenths and eleven-fourteenths of a cwt., and express the sum in lbs

2. Express 9s. 4½d. as the decimal of £1. 7s

3. Subtract ·035 of a guinea from 1·427 of a shilling.

4. Subtract 3·062 of an hour from 1·5347 of a day.

5. Add together ·0029 of a ton and ·273 of a cwt

6. Reduce ·87525 of a mile to feet.

7. Find the sum of 2·35 of 2s. 1d. and $0 \cdot \dot{0}\dot{3}$ of £6. 3s. 9d.

8. Add together $\frac{5}{6}$ of a guinea, $\frac{17}{20}$ of a half-crown, $1\frac{7}{36}$ of a shilling and $\frac{1}{8}$ of a penny, and reduce the whole to the decimal fraction of a pound

9. Express 3s 3d. as the decimal of 10s.

10. Add together $\frac{11}{35}$ of $4\frac{1}{2}$ guineas, £·615 and $3\frac{3}{11}$ of $4\frac{5}{7}$ of $1\frac{5}{9}$ of 16s. 7d.

11. Reduce 49·325 of £4. 12s. 5d. to pence and the decimal of a penny.

12. Add together ·083 of a week and ·231 of a day, and subtract from the sum 8·435 of an hour. Express the result in minutes and the decimal of a minute.

13. What is the difference between ·038 of a mile and $\frac{11}{84}$ of a furlong? Express your answer as the fraction of a furlong.

14. Add together ·0021 of a cwt., ·045 of a quarter, ·37 of a lb., and subtract the sum from 35·263 ounces. Give the answer in ounces and the decimal of an ounce.

15. Add the difference between ·035 of a ton and ·064 of a cwt. to the difference between ·27 of a qr. and ·78 of a lb., and give the answer in lbs. and the decimal of a lb.

16. Add together ·375 of 13s. 4d. and ·07 of £2. 10s., and subtract the result from ·45 of £1.

17. Express 6 cwt. 1·125 qrs. as the decimal of a ton.

18. Reduce £2. 15s. 6¾d. to the decimal of a pound; and find the difference between ·625 and ·0625 of a cwt.

MISCELLANEOUS EXERCISES. XIV.

1. The quotient in a division is 479, the dividend is 3476418 and the remainder is 794; what is the divisor?

2. Subtract $\frac{5}{77}$ of $2\frac{3}{4}$ of £3. 6s. 6d. from $\cdot 0475$ of £100.

3. A gallon contains $277\cdot 274$ cubic inches. Find to three decimal places the number of gallons in a cubic foot.

4. Find the product of $\cdot 052 \times 1\cdot 87 \times \cdot 0021$, and divide the result by the product of $3\cdot 5 \times 6\cdot 63 \times 1\cdot 1$.

5. Find the product of $\cdot 0119$ and $2\cdot 967$. Divide this product by $21\cdot 93$.

6. Find a decimal which shall differ from $\frac{12}{23}$ by less than $\frac{1}{10000}$.

7. Simplify $\frac{1\cdot 5}{\cdot 075} - \frac{3\cdot 25}{1\frac{1}{2}}$.

8. Express as decimals $\frac{6}{11}$ and $\frac{8}{15}$, and subtract one from the other.

9. Divide $\cdot 736$ by $2\cdot 85$ and $2\cdot 85$ by $\cdot 7036$ in each case to four places of decimals, and find the product by multiplying the two quotients together.

10. Multiply $\cdot 1$ by $\cdot 1$, and divide the product by $\cdot 00625$.

11. Simplify $\frac{1\cdot 875}{2\cdot 1} \times \frac{3\cdot 5}{3\cdot 85}$.

12. Find the sum of (i) $2\cdot\dot{4}$, $\cdot\dot{3}\dot{2}$, $\cdot\dot{5}6\dot{7}$, $7\cdot 05\dot{8}$, $4\cdot 1\dot{7}$, $\cdot 4\dot{3}0412\dot{2}$; (ii) $8\cdot 1\dot{9}$, $\cdot\dot{3}0\dot{2}$, $\cdot 502\dot{8}$, $1\cdot 3\dot{0}01$, $\cdot 0\dot{0}6\dot{7}$ and $\cdot 6882036\dot{5}$.

13. Reduce $\frac{5703}{3125}$ to a decimal, and $4\cdot 17\dot{5}4\dot{6}$ to a vulgar fraction in its lowest terms.

14. Convert $\cdot 8\dot{9}\dot{0}$ into a vulgar fraction, and divide $1\cdot 554$ by $\cdot 0037$.

15. Subtract $\cdot 81\dot{7}\dot{9}$ from $5\cdot 63\dot{1}$ without converting them into vulgar fractions.

16. Express the product of $6\cdot 75$ and $\cdot\dot{0}3\dot{7}$ as a vulgar fraction.

17. Find the product of $1\cdot\dot{3}$ and $\cdot 75$; also $2\cdot\dot{6}$ and $\cdot 375$.

18. Express as decimals $\frac{2}{11}$ and $\frac{7}{45}$, and subtract one decimal from the other.

19. Express the quotient of $\cdot\dot{2}\dot{1}$ divided by $\cdot 0\dot{1}\dot{1}$ as a decimal.

20. Express $\frac{10}{37}$ as a decimal and $\cdot 017\dot{3}\dot{5}$ as a vulgar fraction.

21. Reduce $\cdot 6\dot{7}6190\dot{4}$ to a vulgar fraction in its lowest terms.

22. Divide $\cdot 37\dot{5}9\dot{2}$ by $\cdot 05$.

23. Divide $9\cdot 1\dot{6}$ by $\frac{1}{19\cdot 63}$.

24. By how much is the sum of $2\cdot 056$ and $\cdot 92$ greater than their product?

25. Divide $\cdot 03125$ by $\cdot 032$, and $\cdot 032$ by $\cdot 03125$, and multiply the quotients together.

26. Find the product of 1·3 and ·75.

27. What vulgar fractions can be reduced to terminating decimals? Reduce $\frac{3}{125}$ to a decimal.

28. Express the quotient of $\cdot\dot{2}\dot{1}$ divided by $\cdot 0\dot{1}\dot{1}$ as a decimal.

29. Find the sum of $2\cdot\dot{4}$, $\cdot\dot{3}\dot{2}$, $\cdot 5\dot{6}\dot{7}$, $7\cdot 0\dot{5}\dot{6}$, $4\cdot 1\dot{7}$, and $4\dot{3}04122\dot{2}$.

30. Express as a decimal $(\frac{1}{2}+\frac{3}{5}+\frac{5}{7})\div(\frac{5}{2}+\frac{4}{5}+\frac{1}{7})$.

31. What number multiplied by $(\frac{3}{5}+\frac{5}{8}+\frac{1}{16}-\frac{7}{8})$ will give ·37575?

32. Four reams of paper make a pile 21 inches high. Express as a decimal the thickness of a half sheet.

33. Subtract 0·039 of a ton from 1 cwt. 14 lbs.

34. Find the difference in minutes between 0·0375 of a week and 2·857 of an hour.

35. Add together 0·02 of 5 tons, 0·03125 of 8 cwts., and 0·75 of 16 lbs. Express the result as the decimal of 1 ton 15 cwts. 2 qrs. 24 lbs.

36. Add together 0·725 of £1, 7·25 of half-a-crown, and 6·875 of a shilling.

37. If 0·625 of a sum of money is £7. 6s. 8d., what is the whole sum?

38. Add together 0·105 of half-a-crown, 2·07 of a shilling, and 3·21 of a penny, and subtract 0·012 of a sovereign from their sum.

Summary.

To convert a Decimal to a Vulgar Fraction.—Take the given decimal fraction for the numerator, and for denominator write down 1 followed by as many zeros as there are decimal places in the given decimal fraction.

To convert a Vulgar to a Decimal Fraction.—Divide the numerator by the denominator; the quotient when the decimal point is correctly inserted is the fraction required.

Pure Recurring Decimal.—The equivalent vulgar fraction is obtained by writing for numerator all the figures that recur, and for denominator as many nines as there are figures in the recurring decimal.

Mixed Recurring Decimal.—To obtain the equivalent fraction, subtract the non-recurring figures from those which do recur, place the result in the numerator and put in the denominator as many nines as there are recurring figures, followed by as many cyphers as there are non-recurring figures.

CHAPTER V.

SIMPLE ARITHMETICAL METHODS. INVOLUTION. EVOLUTION.

Simple Arithmetical Methods. Many simple labour-saving methods are used by practical men, and these are in most cases suggested by the circumstances of the problem. We may illustrate by one or two simple examples.

To multiply by 5. As $5 = \frac{10}{2}$, add a cipher and divide by 2.

Ex. 1. $736 \times 5 = \frac{7360}{2} = 3680$.

To multiply by 25. Multiply by 100 and divide by 4. The reason for this rule is easy to understand. By adding two ciphers any whole number is multiplied by 100. This result is 4 times greater than is required, so it is divided by 4.

Ex. 2. $52420 \times 25 = \frac{5242000}{4} = 1310500$.

To divide by 25. In a similar manner we proceed to divide by 25 as follows. Multiply the number by 4 and divide by 100.

To divide by 5. $\frac{1}{5} = \frac{2}{10}$, hence to divide by 5, multiply by 2 and divide by 10.

Ex. 3. $27632 \div 25 = \frac{4 \times 27632}{100} = 1105{\cdot}28$.

To divide by 125. Multiply by 8 and divide by 1000. In this case the fact used is that $125 \times 8 = 1000$. If a number is divided by 1000 the result is, therefore, 8 times less than if it is divided by 125. Hence, by dividing by 1000 and then

multiplying by 8, the result is the same as if the number had been divided by 125 directly.

Subtraction. When it is required to subtract one quantity from another, it is often easier to perform the "complementary" operation of addition.

Ex. 4. Subtract 4s. 7½d. from 15s.

In this case we may proceed thus: adding 4½d. to 7½d. we obtain 1s. Mentally this is carried on to the next figure, 4, making it into 5, and to convert 5s. into 15s. we require 10s. Hence, to convert 4s. 7½d. into 15s. we must add 10s. 4½d. In this manner subtracting 4s. 7½d from 15s. is converted into the easier mental operation of addition.

Multiplication and Division. In multiplication and division the contracted methods already referred to will be found very convenient. The work can also often be further shortened by using approximate multipliers.

To multiply by 99. $99 = 100 - 1$; hence to multiply a number by 99, add 2 cyphers and subtract the number.

Ex. 1. Multiply 532 by 99.

$$532 \times 99 = 53200 - 532 = 52668.$$

In a similar manner, to multiply by 999 it is only necessary to add three cyphers and subtract the number.

Ex. 2. Multiply 532 by 999.

Here $\quad 532 \times 999 = 532000 - 532 = 531468.$

Ex. 3. Find the total width in 20 boards, each $11\frac{3}{4}$ inches wide.

Here, instead of multiplying $20 \times 11\frac{3}{4}$, we may, with advantage, multiply 20×12, giving 240, and subtract $20 \times \frac{1}{4}$ or 5,

$$\therefore 20 \times 11\tfrac{3}{4} = 235.$$

Ex. 4. Find the cost of 30 articles at 1s. 7½d. each.

The cost could be obtained by reducing 1s. 7½d. either to pence or half-pence, multiplying by 30, and afterwards reducing to shillings, etc. But the cost can be better obtained mentally, thus:

$$\left.\begin{array}{l} 30 \text{ at } 1\text{s.} = 30\text{s.} \\ 30 \text{ at } 6\text{d.} = 15\text{s.} \\ 30 \text{ at } 1\text{d.} = 2\text{s. } 6\text{d.} \\ 30 \text{ at } \tfrac{1}{2}\text{d.} = 1\text{s. } 3\text{d.} \\ \hline \qquad\qquad 48\text{s. } 9\text{d.} \end{array}\right\} \text{ or total cost} = 48\text{s. } 9\text{d.}$$

INVOLUTION.

Involution. In dealing with the *powers* of numbers (p. 3) we have found that when a number is multiplied by itself once, twice, or more times, the products are called the **powers** of that number; the process by which the powers of numbers are obtained is called *Involution* The number thus multiplied is called the **root**, and the products are called the **powers** of the number. Any number multiplied by the same number is said to be **squared**, and the product so obtained is called the **square** or the **second power** of the original number. The original number is called the **square root** of the product

Thus, $3 \times 3 = 9$ Here the product 9 is the square or second power of 3, and 3 is the square root of 9

It has been explained that instead of writing the expression 3×3 in full, a small figure is placed near the top of the number or quantity, and on the right-hand side of it, thus 3^2. This indicates how many times the number appears in the product. Thus we write 3×3 as 3^2, $3 \times 3 \times 3$ as 3^3, etc The smaller figure written near the top of a number in the manner described is called the **index** or *exponent* of the number.

Adopting this notation, 3^1 would be called the first power of 3, 3^2 the square or the second power, 3^3 the cube or the third power, etc.

The squares, cubes, or even higher powers can be easily obtained if the number is not greater than 10. Thus $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, etc.

The powers of 10 itself are easily remembered, and are as follows:

$$10^2 = 100,\quad 10^3 = 1000,\quad 10^6 = 1{,}000{,}000, \text{ etc}$$

This method of indicating large numbers is very convenient in physical science, in which such numbers as 2, 5, or 10 millions, etc., are of frequent occurrence; for in place of writing 5,000,000 for instance, we may write it more shortly as 5×10^6.

The squares of all the numbers from 1 to 10 are easily remembered; they are as follows:

$$\begin{array}{lll} 1^2 = 1, & 4^2 = 16, & 7^2 = 49, \\ 2^2 = 4, & 5^2 = 25, & 8^2 = 64, \\ 3^2 = 9, & 6^2 = 36, & 9^2 = 81. \end{array}$$

In a similar manner the squares of all numbers from 10 to 20 should be written down.

Ex. 1. Write down the cubes of all the numbers from 1 to 10 inclusive.

EXERCISES. XV.

1. Find $(\cdot 03)^2$.
2. Find the square of $\cdot 00013$.
3. Divide $(\cdot 01)^3$ by $(\cdot 05)^2$.
4. Find the value of $(\cdot 0012)^2 \div (\cdot 02)^3$.
5. Square $\cdot 00625$ and express the result as a vulgar fraction.
6. Find the square and cube of $6\cdot 25$ and of $62\cdot 5$.
7. Find the square and cube of $2\cdot 95$ and of $3\cdot 35$.
8. Divide the cube of $\cdot 29$ by the square of $\cdot 058$.
9. Divide $1\cdot 01364$ by 117 and subtract the square of $\cdot 093$ from the quotient.
10. Find the square of $9\cdot 17$, multiply this by $\cdot 02$, and divide by the product of $17\cdot 161$ and $\cdot 098$.
11. Determine by how much the square of $1\cdot 731$ differs from 3.

Evolution. The reverse of Involution is to extract, or find, the roots of any given numbers.

The root of a number is a number which, multiplied by itself a certain number of times, will produce that number. Thus, the square root of a given number is that number which, when multiplied by itself, is equal to the given number.

The root of a given number may be denoted by the symbol $\sqrt{\ }$ placed before it, with a small figure indicating the nature of the root placed in the angle; in this manner the cube root of 27 is denoted by $\sqrt[3]{27}$, the fourth root of 64 by $\sqrt[4]{64}$, and so on. The square root in this manner would be denoted by $\sqrt[2]{9}$, but the 2 is usually omitted, and it is written more simply as $\sqrt{9}$. Another, and for many purposes a better method, is to indicate the root by a fraction placed as an index, and referred to as a **fractional index**; thus, for example, the square root of 9 is written $9^{\frac{1}{2}}$, and is read as nine to the power one-half. Similarly, the cube root of 27 is written as $27^{\frac{1}{3}}$, meaning 27 to the power one-third.

Square Root.—*Method I.*—To obtain the square root of any quantity in cases where it is not possible to ascertain such root by inspection, we have (to avoid unnecessary repetition) to

adopt a rule. The following example will illustrate the method of extracting a square root.

Ex. 1. Find the square root of 155236.

$$
\begin{array}{rl}
 & 155236\,(\,300+90+4 \\
 & 90000 \\
(2\times 300)+90=690\,) & 65236 \\
 & 62100 \\
2\times 390+4=784\,) & 3136 \\
 & \underline{3136}
\end{array}
$$

The process is as follows:

Divide the given number into periods of two figures each, by putting a point over the *unit's figure*, next on the figure 2 which is in the second place to the left of the 6, and also on the 5, as shown. The given number consists of six figures, the required square root contains three. As $300^2=90,000$ and $400^2=160,000$, the required square root lies between 300 and 400; hence we put 300 to the right of the given number, and subtract its square 90,000; this gives a remainder of 65236.

Put twice 300 to the left of 65236, this divides into 65236 a little over 90 times; add 90 to 2×300, and thus obtain 690; this multiplied by 90 gives a product of 62100; subtract this product from 65236, and the remainder 3136 is obtained.

Next set down to the left of the remainder 3136, $2\times 390=780$; this will divide 4 times into 3136.

Add on 4 to 780, obtaining 784; multiply by 4 and obtain 3136; this subtracted from 3136 leaves no remainder; or 394 is the square root required.

Method II.—The ordinary practical method is as follows:

Point as before, and find the largest number the square of which is less than 15; 3 is such a number. Set the figure 3 to the right of the given number and its square 9 under the first pair of figures 15; subtract 9 from 15, obtaining a remainder 6.

$$
\begin{array}{rl}
 & 155236\,(\,394 \\
 & 9 \\
69\,) & 652 \\
 & 621 \\
784\,) & 3136 \\
 & \underline{3136}
\end{array}
$$

Bring down the next two figures, making the number 652.

Now put the double of 3, that is 6, on the left of the number 652, and by trial find that 6 will divide into 65 nine times. Place the 9 as the second figure of the answer; also put a 9 with the 6 to form the divisor 69 and multiply 69 by 9, thus obtaining 621, which when subtracted from 652 gives a remainder of 31.

Bring down the next two figures, thus obtaining 3136. Double the number 39, that is, the part of the root already found, and put the result 78 to form part of a new divisor, as before.

By trial, we find that 78 will divide into 313 four times. Put the 4 with the other numbers 39 of the square root which is being obtained, and also with the 78, thus making the number 784 the new divisor; this multiplied by 4, the last figure added, gives 3136, which subtracted, leaves no remainder. Hence 394 is the square root required.

The student should always begin to point at the unit's place, whether the given number consists of integers, or decimals, or both.

Ex. 2. Find the square root of 1481·4801.

The pointing begins at the unit's place, and every alternate figure to the right and left of the unit's place is marked as indicated in this example. As there are two dots to the left of the unit's place, the square root consists of the whole number 38 and the decimal; the working is exactly the same as in the previous example.

$$\begin{array}{r|l} & \dot{1}4\dot{8}1\cdot4\dot{8}0\dot{1}\ (\ 38\cdot49 \\ & 9 \\ 68\ ) & 581 \\ & 514 \\ 764\ ) & 3748 \\ & 3056 \\ 7689\ ) & 69201 \\ & 69201 \end{array}$$

It should be carefully noticed that, to obtain the square root of a decimal fraction, the pointing should commence from the second figure of the decimal place.

Ex. 3. Find the square root of ·9216.

$$\begin{array}{r|l} & \cdot9\dot{2}1\dot{6}\ (\ 96 \\ & 81 \\ 186\ ) & 1116 \\ & 1116 \end{array}$$

The method adopted will be evident from the working shown.

As examples, obtain the square roots of the following frequently occurring numbers; these should be worked out carefully, and the first two committed to memory.

$$\sqrt{2} = 1\cdot414\cdots,$$
$$\sqrt{3} = 1\cdot732\cdots,$$
$$\sqrt{5} = 2\cdot236\cdots,$$
$$\sqrt{6} = 2\cdot449\cdots$$

The square root of all these numbers is an unending decimal. Thus, the square root of 3 can be carried to any number of decimal places, but the operation will not terminate. Such a square root is often called a **surd**, or an **incommensurable** number.

In any practical calculation in which *surds* occur, the value is usually not required to more than two or three decimal places.

If a number can be easily separated into factors, the square root can readily be obtained. The method adopted would be to try in succession if the number is divisible by 4, 9, 16, and other numbers of which the square roots are known.

Ex. 4. To find the square root of 1296.

$$1296 = 4 \times 324 = 4 \times 4 \times 81,$$
$$\sqrt{1296} = \sqrt{16 \times 81}$$
$$= 4 \times 9 = 36.$$

A similar method may be adopted in the case of numbers the roots of which cannot be expressed as whole numbers.

Ex. 5.
$$\sqrt{128} = \sqrt{64 \times 2}$$
$$= 8\sqrt{2};$$

and remembering that the $\sqrt{2}$ is 1·414 approximately, the value $8 \times 1{\cdot}414 = 11{\cdot}312$ can be found.

Ex. 6.
$$\sqrt{243} = \sqrt{81 \times 3}$$
$$= 9\sqrt{3}.$$

In many cases where a surd quantity occurs in the denominator of a fraction, it will be found advisable, before proceeding to find the numerical value of the fraction, to transfer the surd from the denominator to the numerator; this is readily effected by multiplication. Thus, if as a result to a given question we obtain the fraction $\frac{100}{\sqrt{3}}$, we may proceed to divide the numerator by $\sqrt{3}$ or 1·732... in order to obtain the numerical value of the fraction; but it is better and simpler to multiply both numerator and denominator by $\sqrt{3}$; this gives $\frac{100\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{100\sqrt{3}}{3}$; and in this form, knowing that $\sqrt{3} = 1{\cdot}732...$, it is only necessary to move the decimal point two places to the right and divide by 3.

Square Root of a Fraction.—In finding the square root of a fraction, it is necessary to obtain the square root of numerator and denominator. When the denominator is not a perfect square, we may proceed to first multiply both the numerator and denominator by the number which will make the denominator a perfect square, or, convert the given fraction to a decimal fraction, and find the root in the usual manner.

Ex. 7. Find the square root of $\frac{7}{8}$.

Here if the numerator and denominator be multiplied by 2, the fraction becomes $\frac{14}{16}$ and its square root is $\frac{\sqrt{14}}{4}$, in this manner only one root requires to be extracted.

Contracted Method. In practical calculations the square root of any quantity is never required to more than a few significant figures, and when more than half the required number of digits have been found, the remainder may be found by contracted division.

Ex. 8. Obtain the square root of 13 to five places of decimals.

Here, proceeding as in the preceding examples, the square root of $13 = 3·605$ is obtained together with a remainder 3975. The remaining figures of the square root may now be obtained by contracted division, viz., by dividing 3975 by 7205, giving 55, which is placed with the number already obtained.

Hence the required root is 3·60555.

```
        13 ( 3·60555
         9
  66 )  400
        396
 7205 ) 40000
        36025
 7205 )  3975 ( 55
         3602
          373
          360
           13
```

EXERCISES. XVI

Extract the square root of

1. 531441.
2. 106929; 803·7.
3. ·191810713444.
4. 13277·9529.
5. ·00015625.
6. 49084036 and of $90\frac{1}{4}$.
7. 91.
8. 502681.
9. (i) 4928·742025; (ii) $\frac{16\cdot9}{225}$.
10. ·0006780816.
11. ·034596.
12. ·05368489.
13. 71.
14. 83·00849881.
15. 49984900.
16. 484·176016
17. 100·020001.
18. 9042049.

19. (i) $\frac{529}{2401}$; (ii) ·081. 20. (i) 123·45; (ii) 234·135.

21. 3249·456016. 22. 25·7049. 23. (i) 8; (ii) ·0002.

24. 19895·1025. 25. 3915380329. 26. $41\frac{184}{225}$.

27. 50013184. 28. 17·375. 29. 1·094116.

30. (i) $20\frac{1}{4}$; (ii) $58\frac{7}{8}$. 31. $32\frac{1}{9}$.

32. If $d = 1·2\sqrt{t}$, where d denotes the diameter of a rivet and t the thickness of plate, find the values of d, when t is (i) $\frac{3}{8}$, (ii) $\frac{9}{16}$, (iii) 1, (iv) $1\frac{1}{2}$.

33. Given $d = \sqrt{330·06 \div 0·7854}$, find the numerical value of d.

34. If $d = \sqrt{0·198 \div 0·7854}$, find the numerical value of d.

35. If $r = \sqrt{1·0744 \div 3·1416}$, find the numerical value of r.

36. If $V = \frac{h}{3}(A + B + \sqrt{A \times B})$, find the value of V, when $h = 10$, $A = 36$, $B = 16$.

Summary.

Involution.—The continued multiplication of a number by itself is called Involution. The number itself is called the first power, the second power is called the square, the third the cube. Thus 2^2 = square of 2, 2^3 = cube of 2, etc.

Evolution.—Given a number, the process of finding the root is called Evolution. The sign $\sqrt{\ }$ indicates the square root. Thus $\sqrt{9}$ means the square root of 9. This is also written $9^{\frac{1}{2}}$.

Cube and Cube Root.—If three equal numbers are multiplied together, the product is called the cube of the number taken, and the number itself is called the cube root of the product. Thus $3 \times 3 \times 3$, or $3^3 = 27$ and $\sqrt[3]{27}$, or $27^{\frac{1}{3}} = 3$.

CHAPTER VI.

AVERAGES. PROPORTION AND RATIO. MEAN PROPORTIONAL. UNITARY METHOD. PERCENTAGES.

Averages.—What is called the mean or average of a series of quantities is often wanted; indeed, the student himself will soon require to use the term average when applied to various values. Thus, for example, if a railway train is found to complete a journey of 60 miles in $1\frac{1}{2}$ hours, we say that the average speed is 40 miles per hour, which means that a train moving uniformly at such a speed would move over 60 miles in the given time.

Again, if in a boat's crew of 8 men the weights of the men are respectively 12, 13, 10, 8, 9, 14, 15, and 11 st; then the total weight

$$=12+13+10+8+9+14+15+11=92 \text{ st.},$$

and the average weight $=\frac{92}{8}=11\frac{1}{2}$ st.

Other simple instances could be quoted, the method in each and all cases being the same. The rule may be stated as follows: To obtain an average, first find the total of the given similar quantities, and divide the result by the number of quantities.

Ex. 1. Find the average value of ·52, ·25, ·32, ·65 and ·48.

Here the total is 2·22, and dividing by the number of terms 5 the average is ·444.

There should be no difficulty experienced in grasping the fundamental principle of averages—namely, *that the sum of the terms is equal to the average multiplied by the number of terms.*

In finding areas and volumes when studying Mensuration, the dimensions given—length, width, diameters, etc.—are assumed

to be *average* dimensions. In this manner errors due to slight irregularities in dimensions are corrected

EXERCISES XVII

1. Divide the mean of 15, 2·01246, 1033 and 82·2 by 10·24.

2. A railway passenger counts the telegraph posts on the line as he passes them. If 24 are passed in one minute, what is the average speed of the train, the distance between each post being 58 yards?

3. If in the next two minutes 37 posts are passed, what is now the average speed?

4. If a truck with its load of 128 packages weighs 8 cwts. 12 lbs., what is the average weight of each package, the weight of the truck being 4 cwts. 1 qr.?

5. What is the average speed of a train (when in motion) which runs from London to Exeter, a distance of $193\frac{3}{4}$ miles, in $4\frac{1}{4}$ hours, making one stoppage of 10 minutes, two of 5 minutes, and one of 3 minutes on the way?

6. In 1881 the population of three towns was 22368, 43415 and 10632 respectively. In 1891 the first had increased by 6 per cent., the second by 8 per cent., and the third decreased 10 per cent. Find the average population of the three towns in 1891.

7. Express as decimals and find the average of $57\frac{7}{20}$, $13\frac{1}{2}\frac{2}{5}$, 21, 0, $7\frac{3}{4}$, $\frac{1}{30}$, $3\frac{1}{8}$ and $106\frac{1}{2}$.

8. A person works for 7 hours on Mondays, 11 hours on Tuesdays, $9\frac{1}{2}$ on Wednesdays, and $11\frac{1}{2}$ on Thursdays, and is paid at the rate of 10d. per hour; find his average daily earnings.

9. Given four measured distances as 113·501, 78·073, 2[illegible]·034 and 9·789 centimetres respectively, find the average and convert to the decimal of a yard.

10. Five sums of money are respectively £2057·85, £3903·73, £1111·95, £1000 and £287·095, find the average.

11. On a voyage lasting $14\frac{3}{4}$ days, the speed of a ship is found to be for the first 3 days at the rate of 13 miles an hour; this speed is increased by 17 per cent. in the next $6\frac{1}{2}$ days, and during the remainder of the journey the speed is 390 miles per day. Find the total distance in miles, and the average speed per hour.

12. If 1 hr. 56 mins. is taken by a train to travel 84 miles 756 yards, what is the average speed per second?

13. The average speed of a train for 2 hrs. 25 mins. is $27\frac{1}{4}$ miles per hour and $43\frac{1}{2}$ miles per hour for 1 hr. 55 mins., find the total distance travelled.

14. Determine the average speed of a train which moves over 120 yards in 15 secs., then 100 yards in 10 secs., and 85 yards in 5 secs.

15. An indicator diagram is divided into 10 equal parts, the pressures of the steam being 180, 180, 135, 100, 80, 63, 57, 50, 44 and 40 : find the average or mean pressure.

16. Find the mean pressure when the given pressures are 100, 100, 100, 71·43, 55·5, 45·45, 38·46, 33·3, 29·41 and 26·31.

17. A man training for a mile race runs the distance every day for 24 days, his time improving at a uniform rate. On the first day he takes 8 mins., on the last $4\frac{3}{4}$. What is his average time?

18. The average weight of the 8 oarsmen in a boat is increased by $2\frac{1}{2}$ lbs. when one of the crew, who weighs 11 st. 12 lbs., is replaced by a new man. What is the weight of the new man?

19. In a procession the route was 7 miles long, and there were on an average 15 rows of spectators on each side; each person occupied 15 inches of frontage. Calculate the number of spectators.

20. How many persons are there in a procession which, moving uniformly at the rate of 3 miles an hour, and containing 10 persons in each yard of its length, takes 80 minutes to pass a place on the route?

Proportion and Ratio compared. In comparing the relative sizes of two objects it is a matter of common experience to refer to one as a multiple,- two or three times, etc., the other; or a sub-multiple - one-half, or one-third, etc., the other; in this manner the comparison is made without reference to the exact size of either. A comparison of the *relative sizes* of two objects, without reference to their *absolute sizes*, gives the idea of **proportion.** Quantities of the same kind are those which may be expressed in terms of the same unit. When two quantities of the *same kind* are considered, the relation which one quantity bears to another is called a **ratio.** Such a comparison is made by considering how many times one quantity is contained in the other; in other words, a ratio is expressed by the quotient obtained by dividing the first quantity by the second.

If the first quantity be 12 and the second 6, there are three ways of expressing the ratio, *i.e.* $\frac{12}{6}$, $12 \div 6$, or, omitting the line, $12 : 6$.

The ratio of 12 things to 6 similar things is definite, and indicates that the number of one kind is twice that of the other; but the ratio of 12 tables to 6 chairs conveys no meaning. In comparing two quantities of the same kind, we can assert that one is twice, three times, or some multiple or sub-multiple of the other, without defining what the unit implies.

Magnitudes may be either abstract or concrete numbers, but the ratio between them must always be abstract. Hence, it is necessary, in comparing magnitudes, that the quantities be written in terms of a common unit. For example, the ratio of 3 tons to 14 lbs., or the ratio of 10 feet to 4 inches is obtained by considering that as there are 2240 lbs. in a ton, the first-named ratio would be $3 \times 2240 : 14$; the second, since 12 inches make 1 foot, would be $10 \times 12 : 4$.

When it is required to divide a number in a given ratio, it is only necessary to add together the two terms of the ratio for a common denominator, and take each in turn for a numerator.

Ex. 1. Divide £35 in the ratio of $2 : 5$. The denominator becomes $2+5$, and the required amounts are $\frac{2}{7}$ of 35 and $\frac{5}{7}$ of $35 = £10$ and £25 respectively.

Beginners are often confused when required to divide a given number in the proportion of two or more fractions, and begin by taking the given fractions, instead of proceeding to reduce to a common denominator. The way to proceed may be shown by an example.

Ex. 2. Divide £70 in the ratio of $\frac{1}{3}$ and $\frac{1}{4}$. This does *not* mean $\frac{1}{3}$ and $\frac{1}{4}$ of 70; but, as fractions with the same denominators are in the same proportion as their numerators, it is necessary to write $\frac{1}{3}$ as $\frac{4}{12}$ and $\frac{1}{4}$ as $\frac{3}{12}$. Then the question is to divide £70 in the ratio $3 : 4$, and the required amounts are $\frac{3}{7}$ of $70 = £30$, and $\frac{4}{7}$ of $70 = £40$.

Proportion.—The two ratios $2 : 4$ and $8 : 16$ are obviously equal, and their equality is expressed either by $2 : 4 = 8 : 16$, or by $2 : 4 :: 8 : 16$. When, as in the given example, the two ratios are equal, the four terms are said to be in *proportion*, hence:

Four quantities are said to be proportional, when the ratio of the first to the second is equal to the ratio of the third to the fourth. That is, when the first is the same multiple or sub-multiple of the second, which the third is of the fourth, the quantities are proportional.

We may thus state that the numbers 6, 8, 15, and 20 form a proportion. The proportion is written as $6 : 8 :: 15 : 20$, and should be read as 6 is to 8 as 15 is to 20.

The first and last terms of a proportion are called the

extremes, and the second and third terms the **means**; in the last example 6 and 20 are the extremes, and 8 and 15 are the means in the proportion.

When four quantities are proportional, the product of the extremes is equal to the product of the means.

Thus, $6\times20=8\times15$, or $\frac{6}{8}=\frac{15}{20}$, in which the proportion is written as the equality of two ratios.

Since the product of two of the terms of a proportion is equal to the product of the other two, it follows at once that if three terms of a proportion are given, the remaining one can be calculated.

Ex. 1. Find the second term of a proportion in which 14, 12 and 15 are respectively the 1st, 3rd and 4th terms.

$$14 : \text{required term} :: 12 : 15;$$

$$\therefore \text{required term} = \frac{15\times14}{12} = 17\tfrac{1}{2}.$$

Direct Proportion. Three quantities are said to be in *direct proportion* when the first is to the second as the second is to the third.

Thus $4:6::6:9$ is a direct proportion.

$$4\times9=6\times6=6^2,$$

$$\text{or}\qquad 6=\sqrt{36}.$$

Mean Proportional.—When the second term of a proportion is equal to the third, each is said to be a *mean proportional* to the other two. Thus, in the above examples 6 is said to be a mean proportional to 4 and 9.

Third Proportional. When three quantities are in proportion and are such that the ratio of the first to the second is the same as the second to the third, then the latter is called a *third proportional* to the other two.

Unitary Method.—By the previous methods of simple proportion, when three out of the four terms are known, we may proceed to find the remaining one. In practice this may often be replaced by a convenient modification called the *Unitary Method*, in which given the cost, or value, of a definite number of articles, or units, we may by division find the value of one

unit, and finally the value of any number of similar units by multiplication. The method may be shown by the following simple example :

Ex. 1. If the cost of 112 articles be 10s., what will be the cost of 212 at the same rate?

Using the three given terms, we may write the following proportion :

$$112 : 10 :: 212 : \text{required term},$$

$$\text{required term} = \frac{10 \times 212}{112} = 18s.\ 11\tfrac{3}{7}d.$$

By the unitary method we should proceed as follows.

If the cost of 112 articles be 10s., then the cost of one article at the same rate is $\frac{10}{112}$s.,

therefore the cost of 212 articles is $\frac{10}{112} \times 212$s. $= 18$s. $11\frac{3}{7}$d.

Geometrical and Arithmetical Means. The *geometrical mean* (written G.M.) between two numbers is found by taking the square root of their product.

Thus, the G.M. of 4 and 9 is $\sqrt{4 \times 9} = 6$; and the G.M. of 9 and 16 is 12.

The *arithmetical mean* (A.M.) is half the sum of two numbers. The arithmetical mean of 4 and 9 is $\frac{4+9}{2} = 6{\cdot}5$. The arithmetical mean of 9 and 16 is 12·5.

Ratios of very small quantities.—In finding the ratio of one quantity to another, it is only the relative magnitudes of the two quantities which are of importance. The quantities themselves may be as small as possible, but the ratio of two very small quantities may be a comparatively large number.

Thus $\frac{1}{1000} = {\cdot}001$ is a small quantity, and so is ·00001, but the ratio of ·001 to ·00001 is $\frac{{\cdot}001}{{\cdot}00001} = 100$. Again ·0000063 is a very small number, and so is ·0000081, but the ratio of $\frac{{\cdot}0000081}{{\cdot}0000063}$ is simply $\frac{81}{63}$ or $\frac{9}{7} = 1\frac{2}{7}$. This very important fact concerning ratio is often lost sight of by beginners, and it must be carefully noted in making calculations.

EXERCISES. XVIII.

1. Find a fourth proportional to 275, 2·75 and ·275.

2. Find the ratio of $23\frac{1}{3}$ of 7 tons 2 cwts. 1 qr. $9\frac{1}{2}$ lbs. to 21 tons 7 cwts.

3. If 100,000 bricks can be obtained for £150, what will be the price of 12?

4. If $35\frac{1}{2}$ lbs. of sugar cost £1. 2s. $2\frac{1}{4}$d., how much will 2 cwts. 51 lbs. cost?

5. If 10 men receive £28. 10s for 6 weeks' work, how many weeks must 8 men work to receive £38?

6. If $\frac{3}{7}$ of an estate be worth £450, what is the value of $\frac{11}{24}$ of the estate?

7. What number bears the same ratio to $\frac{5}{7}$ of 2·40583 that £5. 4s. $3\frac{1}{4}$d. does to £104. 5s. 5d.?

8. Find the ratio which $\frac{2}{3}$ of £27. 1s. $5\frac{1}{4}$d. bears to ·6 of £42. 10s. $10\frac{3}{4}$d.

9. If a train travel 215 miles in 10 hrs. 45 min., what distance will it travel in $24\frac{1}{4}$ hrs. at the same rate?

10. In what time will 25 men do a piece of work which 12 men can do in 15 days?

11. Divide £814 among three persons in the ratios $\frac{2}{5} : \frac{3}{7} : \frac{5}{8}$.

12. If the carriage of 8 cwts. for 120 miles be 24s., what weight can be carried 32 miles, at the same rate, for 18s.?

13. Find a fourth proportional to ·45, ·8, and ·367.

14. Divide 204 into three parts proportional to the numbers 7, 8, 9.

15. Find the number that is to $7\frac{2}{3}$ in the ratio of £3. 1s. 3d. to £4. 13s. 11d.

16. Express the ratios of

(i) $2\frac{1}{4}$ to $7\frac{1}{3}$.

(ii) $\frac{3}{4}$ of 53 cwts. 3 qrs. 3 lbs. to 0·4 of 65 cwts. 0 qrs. 11 lbs.

17. Divide £56 between A, B, C and D in the ratio of the numbers 3, 5, 7 and 9.

18. A sum of £32,818 is to be divided among four persons in the proportion of the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$; find the share of each.

19. A bankrupt owes three creditors £240, £360, and £400 respectively; if £325 is divided among them in proportion to their claims, how much will each receive, and how much will be paid in the £?

20. A gentleman divided the sum of £10 among his three sons in the proportions of their respective ages, which were $12\frac{3}{4}$, $9\frac{3}{4}$, and $7\frac{1}{2}$ years; what was each son's share?

Percentages.—The rate of increase or diminution of one quantity as compared with another of the same kind is often expressed in the form of a percentage. *A percentage is simply a fraction with a denominator of a* 100 This enables a comparison to be made at once, without any preparatory labour, in order to reduce fractions to like denominators. Examples on percentages occur so frequently, and are so varied, that it is difficult to select typical illustrations. The following, however, may make the matter clear.

Suppose that two classes, of 20 and 50 students respectively, are expected to attend an examination. In the first named 18 students, and in the second 47 students, present themselves. Then we might say that 2 in 20 and 3 in 50 were away from the examination; but the comparison is most easily made by finding the percentage in each case Thus, in the first case we have $\frac{2}{20} \times 100 = 10$ per cent.;

in the second case $\frac{3}{50} \times 100 = 6$ per cent.

These results would be written as 10% and 6%.

Ex. 1. Suppose the population of a town in 1885 was 15,990, and in 1890 was 20,550. The actual increase $= 20550 - 15990 = 4560$; but although the actual increase is useful it is much better to be able to state the rate at which the population is increasing for each 100 of its inhabitants. The increase for each 100 of its population is found by simple proportion as follows:

$$15990 : 100 :: 4560 : \text{answer required.}$$

$$\therefore \text{ Answer} = \frac{4560 \times 100}{15990} = 28 \cdot 5$$

Thus the increase for each 100 of its population is 28·5. This number is called 28·5 per cent., and is written 28·5%. The rate per cent. enables an increase or diminution to be readily referred to.

Ex. 2. The population of another town in 1885 was 20,400, and in 1890 was 24,960. The actual increase (as before) is 4560, but it does not follow from this that the two towns are increasing at the same rate. In this case the rate of increase is obtained from:

$$20400 : 4560 :: 100 : \text{answer.}$$

$$\therefore \text{ Answer} = \frac{4560 \times 100}{20400} = 22 \cdot 3.$$

Hence, the population of the latter is not increasing as fast as the former town by 6 per hundred, or, as usually written, by 6%.

In like manner percentages are often used to compare the proportions of lunatics, paupers, criminals, etc., in different towns. Rate or debt collectors and others are usually paid at the rate of so much per cent. If a rate collector is paid at the rate of 2 per cent., for example, this would mean that for every £100 collected he is allowed £2; for every £50, £1, etc.

Ex. 3. If in a machine it is found that a quarter of the energy expended is lost in frictional and other resistances, we should say that 25 per cent. is lost, meaning that $\frac{25}{100}$ is lost. This does not tell us the actual numerical amount of the loss, all that we can infer is that for every 100 units of work expended on the machine 25 units disappear. Such a percentage also enables a comparison to be made, and is a convenient method of expressing the efficiency of machines. If one machine has an efficiency of 75 per cent. and another of 80 per cent., we know that the second is 5 per cent. more efficient than the first.

If, in addition, we know that 25 per cent. is the total loss due to all resistances, but 10 per cent. of this is due to the resistance of a particular part of the mechanism, this gives a *percentage of a percentage* and its numerical value is

$$\frac{25}{100} \text{ of } \frac{10}{100} = \frac{25}{100} \times \frac{10}{100} = \frac{250}{10000} = 2{\cdot}5, \text{ or } 2\tfrac{1}{2} \text{ per cent.}$$

Ex. 4. A reef of quartz contains ·0044 per cent. of gold. If the quartz produces £5. 12s. per ton, find the weight of a sovereign in grains.

$$£5.\ 12\text{s.} = £5\tfrac{6}{10} = 5{\cdot}6 \text{ sovereigns},$$

$$1 \text{ ton} = 2240 \times 7000 \text{ grains},$$

and $\quad \cdot0044 \text{ per cent.} = \frac{\cdot0044}{100} = \cdot000044.$

$$\therefore \text{ weight of } 5{\cdot}6 \text{ sovereigns} = \cdot000044 \times 2240 \times 7000$$

$$= \cdot44 \times 224 \times 7.$$

$$\text{weight of 1 sovereign} = \frac{\cdot44 \times 224 \times 7}{5{\cdot}6}$$

$$= 123{\cdot}2 \text{ grains.}$$

Ex. 5. Seventy-five per cent. of the area of a farm is arable; of the remainder eighty-five per cent. is pasture, and the rest is waste;

the area of the waste is 3 acres 0 r. 20 p. What is the area of the farm?

75 per cent. = $\frac{75}{100}$, or $\frac{3}{4}$ of the area is arable;

25 per cent. = $\frac{25}{100}$, or $\frac{1}{4}$ of the area is pasture and waste;

and of this quarter 85 per cent. is pasture.

15 per cent. is waste.

But $\frac{15}{100} \times \frac{1}{4}$ of the area = 3 ac. 0 r. 20 p.

$\frac{3}{80}$ of the area = 3 ac. 0 r. 20 p.

∴ area of the whole farm = $\frac{80}{3}$ (3 ac. 0 r. 20 p.)

= 83 ac. 1 r. $13\frac{1}{3}$ p.

EXERCISES. XIX.

1. A collector receives 2½ per cent. commission. If he collects £90, find his commission.

2. If oranges are bought at the rate of 12 for 8d. and sold at 8 for 1s., what is the gain per cent.?

3. Find the value of 7¼ per cent. of £125. 4s. 10d.

4. Find the profit per cent. when £7. 1s. 11d. is gained by an outlay of £123. 6s. 8d.

5. Two parts of chicory costing £1. 9s. 9d. per cwt. are mixed with five parts of coffee costing £8. 4s. 6d. per cwt.; the mixture is sold at 1s. 4d. a pound; find the profit per cent.

6. One kind of tea is sold at 3s. a pound, at a profit of 20 per cent.; another kind costs 2s. 8d. a pound. If 4 lbs. of the former are mixed with 5 lbs. of the latter, and the mixture is sold at 3s. 4d. a pound, what is the profit per cent.?

7. A builder sold a house for £945, thereby gaining 8 per cent. on his outlay; what did it cost to build it? If the purchaser lets the house at £70 a year, find how much per cent. per annum he makes on the purchase money.

8. A general having lost two-sevenths of his men in battle, and 6 per cent. of the remainder by sickness, found he had 95,880 men left; how many had he at first?

9. If to every gallon of whisky a spirit merchant adds a pint of water, and sells the mixture at the same price per gallon as he gave for the whisky; what is his profit per cent.?

10. By selling an article for 49s. 6d. the gain is 10 per cent.; what will be the gain if the price is raised to 57s.?

11. By selling 10 acres of land for £4699. 8s. 3d. a man gained 5¼ per cent.; what was the original price per acre?

12. If 17 lbs. of tea worth 4s. be mixed with 25 lbs. at 4s. 8d., and the whole sold at 5s. 4d. per lb.; what is the total gain and the profit per cent.?

13. If gunpowder consists of 10 per cent. sulphur, 15 per cent. charcoal, and 75 per cent. nitre; how much of each material is required to make up $2\frac{1}{2}$ cwts.?

14. If the populations of three towns in 1871 were 42,913, 8724, and 985,577 respectively, and in 1881 it is found that the first has increased by 7 per cent., the second by 10 per cent., and the third by 13 per cent.; find the increase per cent. of the united populations in the same time.

15. From a vessel containing a gallon of water a pint is removed, and in its place a pint of brandy containing 40 per cent. alcohol is added; what percentage of alcohol is there in the gallon of liquid?

16. If eggs are bought at 10s. a gross (144) and sold at 1d. each, what is the profit per cent.?

17. A quantity of ore containing 23 per cent. of copper is bought at 9s. per cwt.; 95 per cent. of the copper is extracted at a cost of 2s. $10\frac{1}{2}$d. per cwt. of ore; find the price per ton at which the copper must be sold if a profit of 15 per cent. is to be made.

18. Two vessels contain each a mixture of water and wine, in the ratio of 2 : 3 and 3 · 7 respectively; what quantity must be taken from each to form a mixture which shall consist of 5 gallons of water and 11 of wine?

19. *A* sells a house to *B* at a loss of 10 per cent.; *B* resells the house to *A* at a gain of 10 per cent.: what percentage of the original cost has *A* gained or lost by the double transaction?

20. In what proportion must a merchant mix one kind of tea at 3s. per lb. with another at 1s. 6d., so that by selling the mixture at 2s. 8d. he may make a profit of 25 per cent.?

21. Two houses are of equal value; by selling one for £127. 10s. there is a loss of 15 per cent. For what sum must the remaining one be sold in order to gain 8 per cent. on the whole transaction?

22. The populations of the upper and lower parts of a town were equal, and after the former had fallen 20 per cent., and the latter risen 15 per cent., the total number of inhabitants was 39,390; what was the population of each part at the first?

23. If the cost of travelling by rail for 42 miles is 5s. 3d., what is the cost of travelling 35 miles at a price per mile 20 per cent. higher?

24. The population of a country is 18,844,000; of these 1,499,000 were employed in agriculture, and 3,110,000 in trades: find the percentages of the whole population of each of these classes.

25. A man sells oranges at 11d. per dozen and so gains 10 per cent. on his outlay. At what price per dozen must he sell them to gain 25 per cent. on his outlay? [U.E.I.]

26. On board a ship there are 656 men, 257 women and 66 children. State these as percentages of the total number of persons. [U.E.I.]

27. (*a*) What is 5 per cent. of 1 ton? (Answer in lbs.) (*b*) Find the number of which 20 is 40 per cent. [L.C.U.]

28. A gun-metal casting weighing 1 cwt. consists by weight of 10% tin and 2% zinc, and the remainder copper. How many lbs. of copper are there in the casting?

State the ratio between the weights of zinc and copper present. [L.C.U.]

29. A piece of work requires 14,250 bricks at 32s per 1000, cartage and mortar at £3 3s 6d, and the labour of 3 men each at 10d. an hour for 5 days of 9 hours each. Twenty-five per cent. is then charged for expenses and profit. Find the price charged. [N.U.T.]

Summary.

Average.—The average or mean of a given number of similar quantities is obtained by dividing the total sum of the quantities by the number of quantities.

Ratio.—The relation between two quantities of the same kind, or the quotient obtained by dividing one by the other, is called a Ratio. Hence the ratio of 3 to 9 $= \frac{3}{9} = \frac{1}{3}$.

Proportion is the equality of two ratios. The four numbers 3, 4, 15 and 20 form a proportion, or the ratio of the first two is equal to the ratio of the last two terms,

$$\frac{3}{4} = \frac{15}{20}, \text{ or } 3 : 4 :: 15 : 20.$$

When three terms are known the fourth can be found.

The product of the means, or 4×15, is equal to the product of the extremes, or 3×20.

Mean Proportional.—When three quantities are in proportion and the second and third terms are equal, each is said to be a *mean proportional* to the other two.

Third Proportional. When the ratio of the first term to the second is the same as the second to the third, the latter is called a *third proportional* to the other two.

Means.—The *geometrical mean* of two numbers is the square root of their product; the *arithmetical mean* is half the sum of the numbers.

Percentage.—By percentage is simply meant a fraction having 100 for its denominator.

CHAPTER VII.

ALGEBRA.

EXPLANATION OF SYMBOLS. SUBSTITUTIONS. ADDITION. SUBTRACTION

Explanation of Symbols.—In dealing with numbers, or digits, as the numerals 1, 2, 3 ... are called, accurate results are obtained whatever be the unit employed. Thus, the digit 7 may refer to 7 shillings, ounces, yards, or other units. In adding two digits, such as 7 and 5, together, we obtain the sum 12, whatever the unit employed may be.

The signs already made use of in Arithmetic are also employed in Algebra, but in Algebra representations of quantities are utilised which have a further generality. Both letters and figures are used as symbols for numbers or quantities. These numbers may be *known* numbers and are then usually represented by the first letters of the alphabet, a, b, c, etc., or they may be numbers which have to be found, called *unknown* numbers, and these are often denoted by x, y, z.

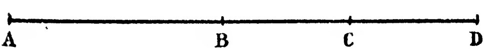

Fig. 4.

A more general meaning is given to the signs + and −.

If a distance AC measured along a line AD (Fig. 4) is said to be positive, the distance CA measured in the opposite direction would be negative.

The result of the first measurement could be indicated by **$+$a**, while the same distance CA, but *measured in the opposite direction*, would be indicated by **$-$a.**

Again, if a length CD be measured in the same direction from left to right and be denoted by **$+$b**, the length DC measured from right to left would be indicated by **$-$b.**

Hence $+a+b$ would mean the sum or addition of the two lines, and so a line of length equal to AD is obtained, where

$$AD = AC+CD$$

Similarly, $+a-b$ would denote the length AB obtained by measuring a length a in the positive and a length b in the negative direction.

The beginner will probably experience some difficulty in the consideration of these positive and negative quantities. In Arithmetic the difficulty is avoided, for the only use that is made of the sign $-$ (Minus) is to denote the operation of subtraction, and in this idea the assumption is made that a quantity cannot be subtracted from another smaller than itself; moreover in Arithmetic we are apt to assume that no quantity is less than 0. In Algebra, on the contrary, we must get the conception of a quantity less than 0, in other words, of a *negative quantity*. Thus, as an illustration, consider the case of a person who neither owes nor possesses anything; his wealth may be represented by 0. Another person who not only possesses nothing but owes £10 is worse off than the first, in fact he is worse off to the extent of £10 compared with the first person. His wealth may, therefore, be denoted by $-$ £10.

Again, in what is called the Centigrade Thermometer the temperature at which water freezes is marked 0°, and that at which water boils 100°, and any temperature between these may be at once written down. But it is often necessary to refer to temperatures below the freezing point. To do this we represent the reading in degrees, but prefix a negative sign to indicate that we measure downwards instead of upwards. Thus $+5°$ C. or, as it is usually written, $5°$ C. indicates five degrees above freezing point, whereas $-5°$ C. indicates five degrees below.

If AC denote a distance of 4 miles in an easterly direction, and AB a distance of $2\frac{1}{2}$ miles (Fig. 4), then a person starting

from A and walking 4 miles in an easterly direction will arrive at C; if when he arrives he proceeds due west a distance equal to $1\frac{1}{2}$ miles he will arrive at B, and his distance from A will be $2\frac{1}{2}$ miles; or if as before a denote the distance AC, and c the distance AB, then if BC be denoted by b, the distance from A would be expressed by $+a-b=+c$.

Algebraical Sum.—When writing down an expression it is usual, where possible, to place the positive quantity first and to dispense with the + sign. The above expressions would, therefore, always be written as $a+b$ and $a-b$. The signs placed between the numbers indicate in the first case the sum of two positive quantities, and in the second case the subtraction of one positive quantity from another. In the latter case the quantity $a-b$ could also be described as the addition of a negative quantity b to a positive quantity a, by which what is called the **algebraical sum** of the two quantities is obtained. The algebraical sum of two or more quantities is, therefore, the result after carrying out the operations indicated by the signs before the quantities.

The algebraical sum of $+10$ and $-18=-8$.

In the quantity $a-b$, if a represents a sum of money received, then $-b$ will represent a sum of money paid away. The algebraic sum is represented by the balance $a-b$.

It will be seen that in Algebra the word sum is used in a different and a wider sense than in Arithmetic. Thus, in Arithmetic $a-b$ indicates that b is to be subtracted from a, but in Algebra it also means the sum of the two quantities.

How a Product is expressed.—The arithmetical symbols of operation, $+$, $-$, $\times$, and $\div$, are used in Algebra, but are varied according to circumstances. The general sign for the multiplication of quantities is $\times$; but the product of single letters may be expressed by placing the letters one after another; thus the product of a and b may be written $a\times b$ but is usually written as ab. In a similar manner the product of 4, a, x, and y is expressed by $4axy$. It will be obvious that this method is not applicable in Arithmetic. Thus 5×7 cannot be written as 57.

The product of two quantities such as $a+b$ and $c+d$ may be expressed as $(a+b)\times(c+d)$, or usually as $(a+b)(c+d)$.

Expression. Quantity.—When, as in $a+b-c$, or $4axy$, several terms are joined together by signs, they form what is called an *algebraic expression or quantity.*

Other names used in Algebra.—Any quantity, such as $4a$, indicates that a quantity a must be taken four times; the multiplier of the letter is called a **coefficient**; and $4a$, containing a coefficient and a letter is called a **term.**

Multiples of the quantities a, b, c, etc., may be expressed by placing numbers before them as, $2a$, $3b$, $5x$; the numbers 2, 3, and 5 thus prefixed are called the coefficients of a, b, and x.

As in Arithmetic, p. 3, the product of a quantity multiplied by itself any number of times is called a **power** of that quantity, and is indicated by writing the number of factors on the right of the quantity and above it. Thus:

$a \times a$ is called the square of a and is written a^2;

$b \times b \times b$ is called the cube of b and is written b^3.

Similarly, $c \times c \times c \ldots n$ factors is written c^n and indicates c to the power n.

The number denoting the power of a given quantity is called its **index** (plural indices) or **exponent**.

It is very important that the distinction between coefficient and index be clearly understood. Thus $4a$ and a^4 are quite different terms. Let $a=2$, then $4a=8$; but $a^4=2^4=16$.

The use of signs may be exemplified in the following manner:

Ex. 1. In the expression a^2+b-c.

Let $a=4,\ b=7,\ \text{and}\ c=3.$

Then $a^2+b-c=4^2+7-3=23-3=20.$

Ex. 2. Find the value of

$$\frac{ax^2+b^2}{bx-a^2-c},\ \text{when}\ a=3,\ b=5,\ c=2,\ x=6.$$

Here $ax^2+b^2=3\times6^2+5^2 \quad =133,$

also $bx-a^2-c=5\times6-3^2-2= \ 19,$

$$\frac{ax^2+b^2}{bx-a^2-c}=\frac{133}{19}=7.$$

Ex. 3. Find the value of

$$(ac-bd)\sqrt{a^2bc+b^2cd+c^2ad-2},$$

when $a=1$, $b=2$, $c=3$, $d=0$.

Substituting these values in the given equation we obtain

$$(3-0)\sqrt{1\times2\times3+4\times3\times0+(9\times1\times0)-2}$$
$$=3\sqrt{6-2}=6.$$

In Ex. 3 it should be carefully noticed that, as one of the given terms d is equal to 0, any term containing that letter must be 0. Hence we may either omit all the terms containing that letter or, by writing them as in the above example, the terms in which the letter occurs are each seen to be equal to zero.

EXERCISES. XX.

1. What signs are used in Algebra to denote addition, subtraction, multiplication and division?

What are the meanings of $-$, $+$, ( )?

2. Represent $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ algebraically when $x=\frac{1}{2}$.

If $a=6$, $b=5$, $c=4$, $d=1$, find the value of

3. $4ab$, $5bc$, $3cd$.
4. c^2+2ab, $2a^3$, $a^2(a+b)$.
5. $a^2+2ab-c+d$.
6. $2a^3-3a^2b+c^3$.
7. $a^2(a+b)-2abc$.
8. $2a\sqrt{(b^2-ac)}+\sqrt{(2ac+c^2)}$.

If $a=5$, $b=3$, $c=4$, $d=2$, $x=7$, $y=5$, $e=1$, $f=0$, find value of

9. $2a-4c+2e+5b-d-x$.
10. $3b+7e-4f+6a-4b-c+3y$.
11. $6b+a+4d-f+3e-6c+2x-3y$.
12. $\dfrac{ax+by}{b+x}$.

Given $V=4$, $t=10$ and $g=32$, find the value of

13. $S=Vt+\frac{1}{2}gt^2$.
14. $S=\dfrac{V^2}{2g}$.
15. $S=Vt-\frac{1}{2}gt^2$.
16. Given $W=75$, $V=110$, find value of $\dfrac{WV^2}{2g}$.

Addition.—The addition of algebraical quantities denotes the expression in one sum of all the like quantities, regard being had to their signs.

When like quantities have the same sign, their sum is found by adding the coefficients and annexing the common letters. Thus $7a+4a=11a$. Also, $7a+3a+3b+5a=15a+3b$.

When several quantities have to be added together, they may be written in columns as here shown; the positive and negative coefficients are then added separately, and the sign of the greater value is prefixed to the common letters. The operation would proceed as follows: Arrange in columns, placing the letters in alphabetical order. Commencing with the row on the left-hand side, we have $7a+3a=10a$. Now add to this $-5a$; or, in other words, from $10a$ subtract $5a$, and the result is $5a$ as shown.

$$\begin{array}{r} 7a+5b \\ -5a+4b \\ 3a-2b \\ \hline 5a+7b \\ \hline \end{array}$$

Again $5b+4b=9b$; and $9b-2b=7b$. Hence the sum required is $5a+7b$.

EXERCISES. XXI.

Add together

1. $a+3b+4c+5d$, $2a-b-3c-6d$, $7a-5b+9c-11d$, and $a+14b+c+23d$.
2. $10a-8b+8c-2d$, $3a+3b-7c+11d$, $-3a-17b+9c+5d$, and $3a+9b-c-12d$.
3. $7a+5b-3c+4d$, $3a-5b-4c-d$, $3b-5a-3c-3d$, $a+b-c-d$, $a-b+c-d$, and $3a-3b+10c+2d$
4. $2x+3a-4b$, $3x+2a-5b$, $4x-8a+7b$, $9x-4a+6b$, $5x+7a-9b$.
5. $3x-7y+2z$, $4y+6z-x$, $-3z-2y+a$, $4x+3z-y$.
6. $-6ax+2by-7$, $3by+18-4z$, $4ax-9-by$, $26+3ax-2by$.
7. $8xy-4az+2bc$, $6az+5mn-3p$, $2bc-3xy+8az$, $-xy-bc-az$, $-4az+3xy-4bc$.
8. $5a+3b$, $-6a+7b$, $a+b+c$, $-a-3b+c$, $2a^2+3a-7b$, $2b-3c+a$.
9. $15a^2+b^2+c^2$, $15a^2+15b^2-15c^2$, $17a^2-17b^2+17c^2$.
10. $2yz+3ab$, $-5cx+2yz+5ab$, $7ab-3cx+yz$, $4ek-6yz-cx$, $2cx+yz-8$.
11. $a^2-3ab+b^2+a+b-1$, $2a^2+4ab-3b^2-2a-2b+3$, $3a^2-5ab-4b^2+3a+4b-2$, $6a^2+10ab+5b^2+a+b$. Find the numerical value when $a=1$, $b=2$.
12. $2a^2-6ab+2b^2+2a+2b-2$, $4a^2+8ab-6b^2-4a-4b+6$, $6a^2-10ab-8b^2+6a+8b+4$, $12a^2+20ab+10b^2+2a+2b$. Find the value of the result when $a=2$, $b=3$

Add together

13. $x^2 - 3xy + y^2 + x + y - 1$, $2x^2 + 4xy - 3y^2 - 2x - 2y + 3$,
$3x^2 - 5xy - 4y^2 + 3x + 4y - 2$, and $6x^2 + 10xy + 5y^2 + x + y$.
Find the numerical value of the result when $x = 2$, $y = 5$.

14. $a^3 + 3a^2b - 6ab^2 + 2b^3$, $3a^3 + 8a^2b + 9ab^2 + 5b^3$,
$-4a^3 - 9a^2b + 4ab^2 - 4b^3$.

15. $a^3 + ab^2 + ac^2 + 5a^2b + a^2c - abc + 3def$,
$2a^3 + 3ab^2 - 3ac^2 + 5a^2b - 7a^2c + 3abc + 3ade$,
$11a^3 - 5ab^2 + 7ac^2 - 3a^2b + 2a^2c - 4abc - 7acd$,
$-a^3 + ab^2 - 2ac^2 - 7a^2b - 2a^2c + 2abc - 3def$.

Subtraction.—In Algebra, to perform the operation of subtraction, arrange the terms as in addition, change the signs of all the terms to be subtracted, and then add to the other expression Thus, to subtract $7a$ from $13a$, we reverse the sign of $7a$ and make it minus; for $13a - 7a$ is only another way of expressing that $7a$ is to be subtracted from $13a$. Thus $13a - 7a = 6a$.

Ex. 1. From $5a + 3x - 2b$ subtract $2c - 4y$. The quantity to be subtracted when its signs are changed is $-2c + 4y$,

$\therefore$ the remainder is $5a + 3x - 2b - 2c + 4y$.

Ex. 2. Subtract $a^2 - 2b - 2c$ from $3a^2 - 4b + 6c$.

Here, after arranging as in addition and changing the sign, we proceed as in addition, thus:
$3a^2 - a^2 = 2a^2$; $2b - 4b = -2b$; and, finally, $6c + 2c = 8c$.

$$\begin{array}{r} 3a^2 - 4b + 6c \\ -\ \ a^2 + 2b + 2c \\ \hline 2a^2 - 2b + 8c \end{array}$$

Hence the result is $2a^2 - 2b + 8c$.

It is not necessary on paper to perform the actual operation of changing the signs of all the terms in the expression to be subtracted. The operation should be carried out mentally.

Ex. 3. From $7x^2 - 2x + 5$ subtract $3x^2 + 5x - 1$.

Here we may as in Ex. 2 write the terms under each other, changing the signs of all the terms in the bottom line, but it is better to write them as they are given. Then after mentally altering the sign of $3x^2$ we obtain by addition $4x^2$. Again mentally altering the sign of $5x$ and adding to $-2x$ we obtain $-7x$; and, finally, repeating the operation for the last figure we get the number 6. Hence the result is $4x^2 - 7x + 6$.

$$\begin{array}{r} 7x^2 - 2x + 5 \\ 3x^2 + 5x - 1 \\ \hline 4x^2 - 7x + 6 \end{array}$$

The subtraction of a negative quantity is equivalent to adding a corresponding positive quantity.

If a length AB, Fig. 5, be denoted by a, and another length BC by b, then $a+b$ would be represented by AC, a line equal in length to $AB+BC$, both being measured in the positive direction (from left to right).

Also $a-b$ would be a quantity obtained by subtracting b from a, and could be obtained by measuring off a length BD in a negative direction, so that $a-b$ is appropriately represented by AD.

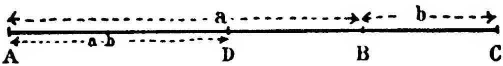

Fig 5

As BC is positive, the reversal of direction indicated by CB is negative, and would be indicated by $-b$. Thus a *minus* sign before a quantity reverses the direction in which the quantity is measured. Now, to subtract b from a, we reversed the direction of b and added it on to a. If then we have to subtract a negative quantity, $-b$ or CB, from a positive quantity a, by reversing the direction we obtain BC or $+b$, and adding on to a we get AC or $a+b$. We could indicate this by $a-(-b)$, the negative sign outside the bracket indicating that the quantity inside the bracket has to be subtracted from a. The change in sign is true whether the quantity subtracted be positive or negative. Hence, the diagram proves the rule already given, namely, to subtract one quantity from another—change the sign of the quantity subtracted and proceed to add the two together.

EXERCISES. XXII.

1. Explain why $-a+b=-(a-b)$.

Subtract

2. $6a^2+a-2b+3$ from $12a^2-3a+b-1$.
3. $5y^2-5y+3a$ from $6y^2-4y-a$.
4. $2a^2+5b$ from $5a^2-2b$.
5. $3xy-8x-7$ from $5xy+8x-2$.
6. $x^2+xy+15$ from $6x^2-xy+10$.

Subtract

7. $4a^3+5a^2b+7ab^2-7b^3$ from $3a^3-5a^2b+6ab^2-7b^3$.

8. $a^3-3a^2b+3ab^2-b^3$ from $a^3+3a^2b+3ab^2+b^3$.

9. $4a-2b+3c$ from $9a+6b-5c$.

10. $5ax+4by-9cz-2$ from $4ax-3by-7cz$.

11. $4x^3-3x^2-x+2$ from $7x^3-6x^2+2x-1$.

12. $3a+2b+c$ from $5a+4b+10c$.

13. $4z+y^2+3x-20$ from $7z+5y+x+10$.

14. $-2mz-3bx+4ay-3$ from $8mz+bx-6ay-2$.

15. $2a^3+3a^2b-4ab^2-7b^3$ from $3a^3-4a^2b+2ab^2-3b^3$.

16. $\frac{3}{4}x-\frac{7}{8}y$ from $\frac{2}{3}x+\frac{3}{5}y$.

17. $3a^2-4ab+5b^2$ from the sum of a^2-ab+b^2 and $2a^2+3ab-4b^2$.

18. Add together $3x^2y^2-10y^4$, $-x^2y^2+5y^4$, $8x^2y^2-6y^4$, $4x^2y^2+2y^4$, and from the sum subtract $10x^2y^2-4x^4-5y^4$.

19. Subtract $2x^3-x^2y-4xy^2+y^3$ from $3x^3-2x^2y+3xy^2-4y^3$.

MISCELLANEOUS EXERCISES. XXIII.

1. Add together $3ab+c$, $4ac-y$, $-3ab-z$, $-2ac-x$, $ac+5y$.

2. Subtract $3ax-2by+4cz+3mn$ from $4ax+4by-7cz+4mn$.

3. Find the value of $(a-b)^2+(b-c)^2+(a-b)(b-c)+5c^2$ when $a=1$, $b=-2$, $c=\frac{1}{2}$.

4. When $x=1$ and $a=-2$, find the value of

$$3(a+2x)^2-2(a+2x)(a-2x)+(a-2x)^2.$$

5. If $a=\frac{2}{3}$, $b=\frac{3}{4}$, $c=\frac{4}{5}$, find the value of

$$(7a-9b)^2+(14b-16c)^2+(9a-12c)^2.$$

6. Find the value of $(5a-3b)(a-b)-b\{3a-c(4a-b)-b^2(a+c)\}$, when $a=0$, $b=-1$, $c=\frac{1}{2}$.

7. $(ac-bd)\sqrt{a^2bc-b^2cd+c^2ad}-2$, when $a=-1$, $b=2$, $c=3$, $d=0$.

8. $3abc-2bcd\sqrt[3]{a^3bc-c^3bd}+3$, when $a=0$, $b=1$, $c=-2$, $d=3$.

9. (i) $20(x-2y)(x+4y)$; (ii) $\frac{4x-8}{3y+1}$;

(iii) $\left(\frac{1}{x}+\frac{1}{y}\right)+\left(\frac{1}{x}-\frac{1}{y}\right)$; (iv) x^2y^3;

when $2x=3$, and $y=-0{\cdot}4$.

10. (i) $a^2x^3y^4$; (ii) $\frac{x}{a-y}-\frac{y}{a+x}$;

(iii) $3xy+4a^2+\sqrt{(10-xy)}$;

when $x=-2$, $y=3$, and $4a=-1$

11. What must be added to $3ax-2x^2$ to make $a^2+ax+3x^2$?

12. From the sum of $2a+3b$, $-6a+7b$, $a+b+c$, $-a-3b+c$, subtract $3a-5b-8c$. Find the value when $a=1$, $b=2$, $c=3$.

13. Add together $x^2+2xy+y^2$, $2x^2-6xy+8y^2$, $x^2-12xy+18y^2$, $4x^2-14xy+13y^2$, $3x^2+18xy-16y^2$, and from the sum subtract $11x^2+24y^2$. Find the value when $x=1$, $y=2$.

14. From $8x^3+4x+3$ take $5x^3-3x^2-4x-13$; then take

$$2x^3-10x^2+8x-16$$

from the remainder. Find the value of the result when $x=1·5$.

15. From the sum of

$$2x^3+ax^2-ax+2a^2x,\quad x^3+10ax^2,\ \text{and}\ 3x^3-5a^2x-8x^3+3ax^2$$

subtract $$2x^3+14ax^2+ax-3a^2x.$$

Find the value of the remainder when $a=2$, $x=4·5$.

16. From the sum of $a+b-3c$, $-a+5b+5c$, and $a-5b$ take the sum of $a+6b+c$ and $5b-4c$. Find the value of the remainder when $a=1·4$, $b=2·5$, $c=3·7$.

17. Simplify $$b+\frac{b+c}{b^2-c^2}-\frac{1}{b+c}.$$

In the result substitute $b=1$, $c=2$, and determine its value.

[L.C.U.]

18. Simplify

$$(a)\ \frac{a^2-b^2}{a+b},\quad (b)\ \frac{a^2+2ab+b^2}{a+b},\quad (c)\ \frac{a^2-ac+ab-bc}{a(a+b)}.$$ [L.C.U.]

19. Find the value of $4a^2+3ab-b^2$ when $a=3$, $b=2$. [L.C.U.]

20. Find the value of y in the expression $y=\frac{10}{1}(2x-4)$ when $x=5$.

21. Find the sum of $\frac{a}{x^2}$, $\frac{4}{x}$, $\frac{3}{x^4}$, $\frac{b}{x^3}$. [L.C.U.]

22. Given $m=11·4$, $n=4·8$, $p=8·6$, find the value of

$$\frac{m(m-p)}{n(p-n)}.$$ [N.U.T.]

23. Find the values of $\frac{a}{b}$, $\frac{a}{bc}$, $\frac{a+c}{c-b}$ and $\frac{abc}{a-b-c}$ if $a=10$, $b=\frac{1}{2}$, $c=8$. [N.U.T.]

24. Find the sum of $6a-b$, $\frac{4b}{3}-\frac{7a}{2}$, $a-\frac{3b}{4}$ and $\frac{a}{5}$. [N.U.T.]

CHAPTER VIII.

ALGEBRA.

MULTIPLICATION. DIVISION. USE OF BRACKETS.

Multiplication.—As already seen in Arithmetic, multiplication may be considered as a concise method of finding the sum of any quantity when repeated any number of times. The sum thus obtained is called the *product*. In Arithmetic the sign $\times$ is employed, but in Algebra this may, or may not be used; thus, $ab = a \times b$; $a(x-y) = a \times (x-y)$, etc.

In multiplying, what is called the Rule of Signs must be observed, *i.e. The product of two terms with like signs is positive; the product of two terms with unlike signs is negative.*

As this rule usually presents some difficulty, we may with advantage explain the reason for it, thus:

Take any two factors, such as 3 and 5, then the possible varieties of the signs may be

$$5 \times 3, \quad -5 \times 3, \quad 5 \times -3, \quad -5 \times -3.$$

The first product we already know is 15, the second is obviously -5 repeated three times and is therefore -15.

The third case is more difficult, but the result may be obtained as follows: Assume the multiplier -3 to be increased by 4, the product would then be 5×4, or 20 *too great*. But -3 increased by 4 is 1, and $5 \times 1 = 5$; hence the result is $5 - 20$, or -15.

In a similar manner, if in the last case the multiplier be increased by 4, the result will be -5×4 or -20 too great. But the multiplier -3 increased by 4 is 1. And since $-5 \times 1 = -5$,

to obtain the product we have to subtract -20 from -5; but as in subtraction the sign of the term subtracted is changed, it is necessary to change the sign of -20, which then becomes $+20$, and the required product is $-5+20=15$.

Similar results would obviously be obtained for the product of any two factors; hence the rule of signs is seen to be true.

In a similar manner, if a is to be multiplied by b, it means that a has to be added to itself as often as there are units in b; hence, the product is ab.

If $-a$ is to be multiplied by $-b$, it means that $-a$ is to be subtracted as often as there are units in b. But since to subtract a negative quantity is the same as to add a positive one (p. 69), the product is again ab.

Again, if $-a$ is to be multiplied by b, it means that $-a$ is to be added to itself as often as there are units in b, hence the product is $-ab$. The same result would be obtained by multiplying a by $-b$.

Rule. To multiply two simple expressions together, *multiply the coefficients and add the indices* (p. 65) *of like letters. Remember also that like signs produce* $+$; *unlike signs produce* $-$.

Ex. 1. $4ab\times 3a^2b^2=12a^3b^3$.

Ex. 2. $4a^3b^2c^5e^4\times 6a^4b^3c^4e^3=24a^7b^5c^9e^7$.

When the expressions consist each of two terms, the process of multiplication is conveniently arranged as follows.

Ex. 3. Multiply $x+5$ by $x+6$.

Write down the two expressions as shown, one under the other; multiply each term of the first expression by each term of the second, and arrange the results as here indicated; begin at the left-hand side, thus, $x\times x=x^2$. Write the x^2, and the product of x and 5 or $5x$. As the signs are alike, the sign of each of these products is $+$. Next multiplying by the second term 6 we get $6x+30$; the term $6x$ is placed immediately below the corresponding term $5x$; and the term 30 on the extreme right-hand side; finally, add the terms together to obtain the product. By arranging the terms one under the other, and multiplying, the result can always be obtained. But this is not enough; in such a simple expression the student should be able to at once write down the product by inspection.

$$\begin{array}{r}x+5\\ x+6\\ \hline x^2+5x\\ 6x+30\\ \hline x^2+11x+30\\ \hline\end{array}$$

This is effected by noting that the first term x^2 of the product is obtained by multiplying together the two first terms in the given expressions; the last term is the product of the two second terms 6 and 5, and the middle term is the product of x and the sum of the two second terms.

$$\therefore \quad (x+5)(x+6) = x^2 + 11x + 30.$$

In a similar manner,

$$(a+b)^2, \text{ or } (a+b)(a+b) = a^2 + 2ab + b^2,$$
$$(a-b)^2, \text{ or } (a-b)(a-b) = a^2 - 2ab + b^2,$$
$$(x-5)(x-6) = x^2 - 11x + 30,$$
$$(x-5)(x+6) = x^2 + x - 30,$$
$$(x+5)(x-6) = x^2 - x - 30.$$

When the product of two expressions containing more than two terms is required, it is usually convenient to arrange the terms one under the other, and to proceed as in the following example.

Ex. 4. Multiply together $14ac - 3ab + 2$ and $ac - ab + 1$.

$$\begin{array}{llllll}
14ac & -3ab+2 & & & & \\
ac & -ab+1 & & & & \\
\hline
14a^2c^2 & -3a^2bc & +2ac & & & \\
 & -14a^2bc & & +3a^2b^2 & -2ab & \\
 & & +14ac & & -3ab & +2 \\
\hline
14a^2c^2 & -17a^2bc & +16ac & +3a^2b^2 & -5ab & +2
\end{array}$$

Proceeding as in Ex. 3 we have

$$14ac \times ac = 14a^2c^2.$$

Next $\qquad -3ab \times ac = -3a^2bc,$

and finally $\qquad ac \times 2 = 2ac.$

In a similar manner, by multiplying by $-ab$ the second line is obtained. After writing down the third line, the terms are added and the product is thus found.

Continued Product. When several quantities are multiplied together the product obtained is called the *continued product* of the quantities.

Ex. 1. The continued product of $3b$, $7c$ and $2a$ is $42abc$.

EXERCISES. XXIV.

Multiply

1. $a^3 + 2a^2b + 3ab^2 + 3b^3$ by $a^3 - 3a^2b + 3ab^2$.
2. $x^2 - 4xy + 6y^2$ by $x - 5y$. 3. $x^2 - 5xy + 6y^2$ by $x - 4y$.
4. Prove that $(a-b+c)(b-c+a)-(c-b+a)(c+a+b)=2c(b-c-a)$.

Multiply

5. $4a^2+12ab+9b^2$ by $4a^2-12ab+9b^2$.

6. $5x^5-3x^3+x$ by $4x^4-2x^2+5$. Find the numerical value when $x=-1$.

7. $(a^2+b^2+c^2-ab-ac-bc)$ by $(a+b+c)$.

8. Show that $(-a)\times(-b)=+ab$.

9. Multiply $5x^2-6xy+7y^2$ by $7x^2+6xy-5y^2$.

10. Express by algebraical symbols: Three times the square of a multiplied by b added to the difference between the cubes of a and b.

11. Multiply $x^2+y^2+z^2+xy-xz+yz$ by $x-y+z$.

12. Multiply together x^2+ax+a^2, x^2-ax+a^2, and x^2-a^2.

Multiply

13. $a^2+2ab+b^2-c^2$ by $a^2-2ab+b^2-c^2$.

14. $x^2+y^2+z^2-yz-zx-xy$ by $x+y+z$.

15. $4a^2-4ab-3b^2$ by $2a^2+3ab-9b^2$.

16. $\frac{3}{2}x^2-x+\frac{2}{3}$ by $3x^2+2x+1\frac{1}{3}$.

17. $a-2b+1$ by $2a+b+1$, and by $2a+3b+1$.

18. Multiply together x^2-7x+6, $x^2+7x-18$, x^2-1.

Multiply

19. $a^2+2ab-3b^2$ by $a^2-5ab+4b^2$.

20. $x^3-ax^2-2a^2x+a^3$ by x^2+ax-a^2.

Division.—In Algebra, as in Arithmetic, the terms **divisor, dividend** and **quotient** are used; hence, from a given dividend and divisor, we can by the process of division, proceed to find the quotient of two or more algebraical expressions. When the divisor is exactly contained in the dividend, then the product of the divisor and the quotient is obviously equal to the dividend. When the divisor is not exactly contained in the dividend, and there is a remainder, the remainder must be added to the product of the quotient and the divisor in order to give the dividend.

Ex. 1. Divide $18ax^2$ by $9ax$;

$$\therefore \frac{18ax^2}{9ax}=2x.$$

As in Arithmetic the work may be done by cancelling, thus,

$$18\div 9 \text{ gives } 2,$$

and $$ax^2\div ax \text{ gives } x,$$

hence the required quotient is $2x$.

Ex. 2. Divide $15a^2b^2$ by $-5a$;

$$\frac{15a^2b^2}{-5a} = -3ab^2.$$

In each case by multiplying divisor and quotient together we obtain the dividend

When the dividend and divisor both consist of several terms, we arrange both dividend and divisor according to the powers of the same letter, beginning with the highest. The following example worked out in full will show the method adopted:

$$\begin{array}{l}
a^2+2ax+x^2\,)\,a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5\,(\,a^3+3a^2x+3ax^2+x^3 \\
\qquad\qquad\quad \underline{a^5+2a^4x+a^3x^2} \\
\qquad\qquad\qquad\quad 3a^4x+9a^3x^2+10a^2x^3 \\
\qquad\qquad\qquad\quad \underline{3a^4x+6a^3x^2+3a^2x^3} \\
\qquad\qquad\qquad\qquad\qquad 3a^3x^2+7a^2x^3+5ax^4 \\
\qquad\qquad\qquad\qquad\qquad \underline{3a^3x^2+6a^2x^3+3ax^4} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad a^2x^3+2ax^4+x^5 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{a^2x^3+2ax^4+x^5}
\end{array}$$

Divide the first or left-hand term of the divisor into the dividend. Thus, a^2 into a^5 gives a^3; write this quantity on the right-hand side as shown, and put the term a^5 under the first term of the dividend. In a similar manner by multiplying the remaining two terms $2ax$ and x^2 by a^3 and subtracting we obtain $3a^4x+9a^3x^2$. Now bring down the next term $10a^2x^3$, and proceed as before.

EXERCISES. XXV.

Divide

1. $x^2+2xy+y^2$ by $x+y$.
2. $a^3+5a^2x+5ax^2+x^3$ by $a+x$.
3. $x^3-9x^2+27x-27$ by $x-3$.
4. a^3-b^3 by $a-b$.
5. a^3+b^3 by $a+b$.
6. $a^3-4a^2x+4ax^2-x^3$ by $a-x$.
7. $a^4+a^2b^2+b^4$ by a^2+ab+b^2.
8. a^4+4b^4 by $a^2-2ab+2b^2$.
9. $x^6-2a^3x^3+a^6$ by $x^2-2ax+a^2$.
10. $a^4-a^2b^2-a^2c^2+b^2c^2$ by $a^2-ab+ac-bc$.
11. $x^6-20a^3x^3+343a^6$ by $x^2+ax+7a^2$.
12. $6x^4+5x^3y+6x^2y^2+5xy^3+6y^4$ by $2x^2+3xy+2y^2$.
13. x^6-a^6 by x^2-ax+a^2.
14. $x(x^2-yz)+y(y^2-xz)+z(z^2-xy)$ by $x+y+z$.
15. $2x^5-10x^4y+9x^3y^2+13x^2y^3-18xy^4+3y^5$ by $2x^2-3y^2$.

Divide

16. $x^6 - (p^4 - 2pq)x^4 - p^2q^2x^2 - q^4$ by $x^3 - p^2x^2 + pqx - q^2$.
17. $(x+y)^5 - x^5 - y^5$ by $x^2 + xy + y^2$.
18. $x^5 + x^4 + 4x^3 + 21x^2 + 23x - 40$ by $x^2 + 4x + 5$.
19. $x^6 - a^6$ by $x^2 - ax + a^2$.
20. $x^5 + x^4 - 9x^3 + 33x^2 - 7x - 49$ by $x^2 + 3x - 7$.
21. $2x^3 - 5x^2y - 9y^3$ by $x - 3y$.
22. Obtain the first six terms in the quotient when $1 - x + x^2$ is divided by $1 + x$.

Divide

23. $3x^3 + 4abx^2 - 6a^2b^2x - 4a^3b^3$ by $x - 2ab$.
24. $a^5 + a^3 - a^3b^2 - a^2b - ab^2 + b^3$ by $a^2 + a - b$.
25. $14x^4 + 45x^3y + 78x^2y^2 + 45xy^3 + 14y^4$ by $2x^2 + 5xy + 7y^2$, and verify the result by putting $x = 1$, $y = 2$ in the divisor, dividend, and quotient.

Use of Brackets.—In Algebra it is frequently necessary to group parts of an expression, and the use of brackets for this purpose is very important. There are several forms of brackets in general use; for example, ( ), { }, [ ]. Sometimes a line is placed over two numbers, and such a line has the same meaning as enclosing in brackets. Thus, if a quantity $b+c$ has to be multiplied by $d+f$ the terms may be written as $\overline{b+c} \times \overline{d+f}$, or as $(b+c)(d+f)$. In this way the use of brackets gives a short method of indicating multiplication. The use of the different forms of brackets can be shown by the following examples.

Ex. 1. $3a - (4b - 7c)$.

Here, the brackets indicate that $4b - 7c$ is to be subtracted from $3a$, and it is obvious that the result obtained will be the same whether we first subtract $7c$ from $4b$ and afterwards subtract the remainder from $3a$, or first add $7c$ to $3a$ and subtract $4b$ from the sum.

A positive or negative sign may occur before a bracket: if the former, then the signs of all the terms are unaltered when the brackets are removed; if the latter (or negative sign), the signs of each term inside the brackets must be changed.

***Ex.* 2.** $3a + (4b - 7c + 3d) = 3a + 4b - 7c + 3d.$
$3a - (4b - 7c + 3d) = 3a - 4b + 7c - 3d.$

The other forms of brackets which are used are [ ] and { }. In each case they denote that whatever is in one pair of them is

to be regarded as one quantity to be added, subtracted, multiplied or divided as a whole in the manner which the signs and quantities outside the brackets indicate.

Ex. 3. Express the product of $2a+3b$ and $4c+5d$.
The quantities may be written as $(2a+3b)(4c+5d)$.

Further, to indicate that $3f$ is to be subtracted from the product and the result multiplied by $7e$, we use another pair of brackets, thus, $7e\{(2a+3b)(4c+5d)-3f\}$; and to express that when $3x$ is subtracted from the last obtained product the whole must be multiplied by 8, we have to use another bracket, thus,

$$8[7e\{(2a+3b)(4c+5d)-3f\}-3x].$$

In removing the brackets it will be found best to start from the inside pair. Moreover, to prevent mistakes, it is advisable only to remove one pair of brackets at each step.

Ex. 4. Simplify

$$4x-[(4x-4y)(4x+4y)-\{4x+(4x+4y)(4x-4y)\}+4y]$$

Multiplying the terms in the brackets we get

$$\begin{aligned} &= 4x-[16x^2-16y^2-\{4x+16x^2-16y^2\}+4y] \\ &= 4x-[16x^2-16y^2-4x-16x^2+16y^2+4y] \\ &= 4x-16x^2+16y^2+4x+16x^2-16y^2-4y \\ &= 8x-4y. \end{aligned}$$

EXERCISES. XXVI.

1. Explain the use of brackets, and prove that

$$a+(b-c)=a+b-c,$$
$$a-(b-c)=a-b+c.$$

2. Prove that $a-\{b-(c-d)\}=a-b+c-d$.

Simplify

3. $5(a-b)-2\{3a-(a+b)\}+7\{(a-2b)-(5a-2b)\}$.
4. $2(3x-y)-4\{2x-(x-y)\}-3\{(x-4y)+2(3x-y)\}$.
5. $a-3[b-2\{a-3b\}-2a]$.
6. $3(x+z)-(6y-z)-2\{x-(2y+z)-(y-3z)\}$.
7. $1+x-[1+x+\{1-x-(1+x)-(1-x+1)\}]$.
8. $(a-b-c)+(b+c-d)-(c-d-f)-(f+g-c)$.
9. $3a-[a+b-\{a+b+c-(a+b+c+d)\}]$.
10. $3a+5b-2c-\{3a-2b-(7b-a+3c)\}$.
11. $1-[2-(7-\overline{x-4})]+2-[3+(4-\overline{x-5})]$.

Simplify

12. $13a - [7a + b - \{11a - 5b - (17a + b)\}]$.

13. $\dfrac{a}{4} - \left[\dfrac{2c - 3a}{4} - \left\{c - \dfrac{a - 2c}{2} - \left(2c - \dfrac{3a + c}{2}\right)\right\}\right]$.

14. $3(2a - b - c) - 5\{a - (2b + c)\} + 2\{b - (c - a)\}$.

15. Find the value of $\dfrac{4(x+1)}{3} - \{2x - (x - 3)\}$ when $x = 37$.

16. Find the numerical values of the following expressions when $x = -1$, $y = -2$, $z = \frac{1}{2}$.

(i) $2x - \{9y - 8x + 2z - (4x + y)\}$;

(ii) $(x + y - z)^2 + (x + y)^2(x - y + z) + (x - y)^3$.

Simplify

17. $7a + 5b - 2c - (4a - 3b - 2c)$.

18. $a + 2b - 3c - \{a - 2b - (a + b - c)\}$.

19. $7a - [4a + (3a - b)]$.

20. $4\{3a - 2(b - c + a)\} - [6a - \{2c - (5b - 4a)\}]$.

21. $[7a - 3b - 4\{2b - 3a - (4b - c)\}] - [c - 2(3a - b) - 5\{2a - (6b - c)\}]$.

22. $(a + b + c)^3 - (a + b - c)^3 + (a - b + c)^3 + (-a + b + c)^3 - 24abc$ when $2a = 2 = -2b = c$.

23. $\dfrac{1}{x - 2a} + \dfrac{2}{x + b} + \dfrac{1}{b}$ when $x = \dfrac{3ab}{b - a}$.

24. $\dfrac{3(x + y + z)(yz + zx + xy) - x^3 - y^3 - z^3}{x^2 + y^2 + z^2 - yz - zx - xy}$ when $x = \frac{1}{2}$, $y = \frac{2}{3}$, $z = \frac{3}{4}$.

25. When $a = 1$, $b = 3$, $c = 4$, $d = 0$, find the value of

(i) $3a^2b - 5[b^2c + 2(b - 3c)] + ab(b^3 - cd) + \dfrac{2a}{3}$.

(ii) $\dfrac{\sqrt{b^2 + c^2}}{a + 2b + c} + \dfrac{5(a + 2c)}{b - d}$.

(iii) $3ab^2 - 4\{bc^2 + 2(c - 3b)\} + ac(b^3 - dc) + \dfrac{2b}{3}$.

(iv) $\dfrac{\sqrt{a + 2c}}{a + 2b + d} + \dfrac{3(a + 2b)}{2c - a}$.

MISCELLANEOUS EXERCISES. XXVII

1. Multiply $3x^3 - 2ax^2 - 4a^3$ by $2x^2 - 7ax + a^2$.
2. Divide $x^4 + 2x^3 + 1$ by $x^2 + 2x + 1$.
3. Show that $x(x - 1)(x - 2)(x - 3) + 1 = (x^2 - 3x + 1)^2$.
4. Divide $6x^4 + 5x^3 + 6x^2 - 17x + 6$ by $6x^2 - 7x + 2$.

5. Add together

$24\left\{\frac{2x}{3}-\frac{1}{4}\left(\frac{5y}{3}+\frac{z}{6}\right)\right\}$, $16\left(\frac{3x}{8}-\frac{2y-3z}{1}\right)$ and $20\left\{\frac{3z}{10}-\frac{3}{5}(2x-y)\right\}$.

6. (i) Multiply $x^2-5xy+6y^2$ by $x-4y$;

(ii) Divide $4x^3-3x^2y-20y^3$ by $x-2y$.

Find the numerical value of

7. (i) $8a^3+27b^3+c^3-18abc$;

(ii) $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$;

when $6a=1$, $9b+1=0$, and $2c=1$.

8. $1680+(r-7)[1470+(r-8)\{378+(r-9)(35+r-10)\}]$, when $r=-7$.

9. Remove the brackets from

$$(3a-5b)(a-c)+c\{2a-c(3a-b)-b^2(a-c)\},$$

also find its value when $a=0$, $b=1$, $c=-\frac{1}{2}$.

10. Remove the brackets from

$$7a+6[b-5\{c+4(b-3)(a+2c)\}],$$

and find its value when $a=2$, $b=3$, and $c=1$.

Add together

11. (i) $\frac{x^3}{3}-\frac{x^2y}{4}+\frac{xy^2}{4}-\frac{y^3}{3}$, $\frac{x^3}{2}+\frac{x^2y}{12}-\frac{xy^2}{12}+\frac{y^3}{2}$;

(ii) $4a^3+b(3c^2-6a^2)+5a^2b-c(c^2+3bc)+c^3-2a^2(a+b)$.

12. $24\left\{\frac{2x}{3}-\frac{1}{4}\left(\frac{5y}{3}+\frac{z}{6}\right)\right\}+16\left(\frac{3x}{8}-\frac{2y-3z}{1}\right)+20\left\{\frac{3z}{10}-\frac{3}{5}(2x-y)\right\}$

13. $4b^3+c(3a^2-6b^2)$, $5b^2c-a^2(a+3c)$, $a^3-2b^2(b+c)$.

14. $18\left\{\frac{2x}{9}-\frac{1}{6}\left(\frac{2y}{3}+z\right)\right\}$, $24\left(\frac{3x}{8}-\frac{2y-3z}{12}\right)$, $30\left\{\frac{7z}{15}-\frac{4}{5}(2x-y)\right\}$.

Multiply

15. $a+b+\frac{b^2}{a}+\frac{a^2}{b}$ by $a-b+\frac{b^2}{a}-\frac{a^2}{b}$.

16. $\frac{x^3}{2}-\frac{5x^2}{3}+\frac{x}{12}+3$ by $\frac{x^2}{2}-x+3$.

Subtract

17. $(a-b)x-(b-c)y$ from $(a+b)x+(b+c)y$.

18. $\frac{x^3}{3}-\frac{x^2y}{4}+\frac{xy^2}{4}-\frac{y^3}{3}$ from $\frac{x^3}{2}+\frac{x^2y}{12}-\frac{xy^2}{12}+\frac{y^3}{2}$.

19. $3y-x+z$ from $3x-5y-6z+(2x+3y+4z)-(4x-3y-3z)$.

20. $b^2-\frac{3a^2}{5}-\left(\frac{8}{9}c^2-\frac{6}{7}d^2\right)$ from $a^2-\frac{b^2}{4}-\left(3d^2-\frac{c^2}{9}\right)$.

Subtract

21. $a(2b+c)-3bc$ from $a(b+2c)-3bc$.

22. $b(2c+a)-3ca$ from $b(c+2a)$

23. $\left(a-\frac{b}{2}+c\right)$ from $a+\frac{b}{2}+\frac{a}{3}+c$

24. $3x^3-x^2-x-7$ from $4x^3+2x^2+x+1$ and from the remainder take x^3-4x^2+2x+8.

25. Find the value of

$$\frac{a+b}{ab}(a^2+b^2-c^2)+\frac{b+c}{bc}(b^2+c^2-a^2)+\frac{c+a}{ac}(c^2+a^2-b^2),$$

when $a=3$, $b=4$, $c=-5$.

26. Find the value of

$$\frac{(ac-b)(a+b)-bc(c-a)-ca(a-b)}{(b-c)(c-a)(a+b)},$$

when $a=1$, $b=3$, $c=4$

27. Find the numerical value of

$$\frac{3a-4b}{6a-5b}-\sqrt{\frac{4a}{a-2b}},$$

when $2a=1$ and $b=-2$

28. Find the numerical value of

$$\frac{a-b}{a+b}+\sqrt{\left(2-\frac{a}{b}-\frac{b}{a}\right)},$$

when $8a=1$ and $2b=1$

29. Simplify

$$(a-b)(b-c)(c+a)+(b-c)(c-a)(a+b)+(c-a)(a+b)(b+c),$$

and find its value when $a=1$, $b=3$, and $c=-2$.

30. Find the value, when $x=5$ and $y=3$, of

$$\frac{x^4-4x^3y+6x^2y^2-5xy^3+2y^4}{2x^4-5x^3y+6x^2y^2-4xy^3+y^4}.$$

31. If $x=2$ and $y=-\frac{1}{2}$, find the numerical values of

$$3(x+y)-2(x-y),\quad x^4y^6,\quad x^3y^3,\quad \text{and}\quad \frac{x^3-y^3}{x-y}.$$

32. Multiply $-6a^4xy^2$ by $10ax^2y^3$ and divide the product by $15a^3xy^4$. Verify the result by substituting $a=1$, $3x=2$, and $y=-3$, in the multiplicand, multiplier, divisor, and quotient.

33. (i) Simplify $2a-[x^2-\{3y-a-(2x-x^2)\}-(a-2y)-3x]$;

(ii) Find the numerical value of

$$[(p-q)^2+(q-r)^2+(r-p)^2]\div(p^2+q^2+r^2),$$

when $p=-2$, $q=-3$, $r=-4$

34. Subtract the sum of

$a^3 - x^2 - \{a - x - (2x^2 - 4)\}$ and $a^3 - x^3 - \{a + x - (a^2 - x^3 - 3)\}$
from $2x^2 - x^3$.

35. Find the value of

$$\frac{\sqrt{a^2+b^2+c^2}}{a-2b(a-3c+2)}+\frac{\sqrt[3]{b-a}}{3c+\frac{1}{2}}$$

when $a=7$, $b=-1$, $c=\frac{5}{3}$.

Express the following processes algebraically:

36. Square a, divide the result by the square of b, then add 1 and extract the square root. Multiply the square root by a and divide by the square of a.

37. x is multiplied by the fourth power of y, and this product is subtracted from the square of x multiplied by the cube of y; the cube root of the square of this difference is divided by the square root of the sum of x and y.

38. Multiply the cube of x by the square of y. Multiply the square of x by the cube of y; subtract the second from the first; extract the square root and multiply by the sum of $5x$ and $3y$.

39. Multiply the square of $(a-b)$ by $5a$. [N.U.T.]

40. Add $2a+3b$, $3a-5b$ and $2b-a$; and multiply the result by $a+b$. [N.U.T.]

41. Divide $15a^3b^3 - 10a^2b^4 + 5ab^5$ by $5ab^3$. [N.U.T.]

42. Divide $\frac{3}{4}x^2 - \frac{4}{3}y^2$ by $\frac{x}{4}-\frac{y}{3}$. [N.U.T.]

Summary.

Multiplication.—The product of two terms with like signs is positive, that of two terms with unlike signs is negative. To multiply two simple expressions together, multiply the coefficients and add the indices of like letters.

Division.—The object of division is usually to find how often one quantity (the divisor) is contained in another (the dividend); the product of the divisor and the quotient, plus the remainder, if there is one, is equal to the dividend.

Brackets.—The forms of brackets in general use are ( ), { }, and [ ]; they are used to indicate that whatever is in one pair of them is to be regarded as one quantity. Brackets are usually removed in the order named, beginning with ( ).

CHAPTER IX.

SYMBOLICAL EXPRESSION. SIMPLE EQUATIONS. PROBLEMS PRODUCING SIMPLE EQUATIONS.

Symbolical Expression. One of the greatest difficulties experienced by the beginner in Algebra is to express the conditions of a given problem by means of algebraical symbols, and some practice may be necessary before even the simplest problem can be stated. The few examples which follow are typical of a great number.

Let x denote a quantity; then 5 times that quantity would be $5x$, the square of that quantity would be x^2, and a fourth part of the quantity would be indicated by $\frac{x}{4}$.

If a sum of £50 were equally divided among x persons, then each person would receive $\frac{£50}{x}$.

If the difference of two numbers is 7, and the smaller number be denoted by x, the other will be represented by $x+7$. If the larger be denoted by x, then the smaller will be represented by $x-7$.

If the distance between two towns is a miles, the time taken by a train travelling at x miles an hour would be $\frac{a}{x}$; when the numerical values of a and x are known, the time taken can be obtained. Thus, let the distance a be 200 miles, and x the velocity, or speed, be 50 miles an hour, then the time taken is

$$\frac{200}{50}=4 \text{ hours.}$$

Although the letters a, x, etc., are used in algebraical operations, symbols are often employed which at once, by the letters used, express clearly the quantities indicated. Thus, distance or space could be denoted by s; the speed, or velocity, by v; and the time taken by t; then instead of $\frac{a}{x}$ we use $\frac{s}{v}$, or the relation between s, v, and t is given by $s = vt$. From this relation, when any two of the three terms are given, the remaining one may be obtained.

In the case of a body falling vertically, the relation between space described and time of falling is given by $s = \frac{1}{2}gt^2$; where s denotes the space described in feet, t the time in seconds, and g denotes 32·2 feet per second, or the amount by which the velocity of a body falling freely is increased in each second of its motion. In this case, given either s or t, the remaining term may be found.

Equations. An equation may in Arithmetic, or Algebra, be taken simply to be a statement that two quantities are equal.

Thus, the statement that 2 added to 7 is 9, may be expressed as an equation by $2 + 7 = 9$. Obviously, in a similar manner, other statements of equality, or, briefly, other equations, could be formed; and it will at once be evident that the greater part of the student's work in Arithmetic has been concerned with such equations.

All such equations, involving only simple arithmetical operations, may be called *Arithmetical Equations*, to distinguish them from such equations as $2x + 7 = 9$, which is called an *Algebraical Equation*. As in Arithmetic, the answer to any given question remains unknown until the calculation is completed, so in Algebra the *solution of an equation* consists in finding a value, or values, which at the outset are unknown.

Simple Equations.—When two algebraical expressions are connected together by the sign of equality, the whole expression thus formed is called an *equation*, and the use of an equation consists in this, that from the relations expressed between certain known and unknown quantities we are able under proper conditions to find the unknown quantity in terms of the known.

Usually the earlier letters of the alphabet, a, b, c, d, ..., are

used to represent known and its concluding letters, x, y, z, to represent unknown quantities.

The process of finding the value of the unknown quantity is called **solving the equation**; the value so found is the **solution** or the **root** of the equation. This root, or solution, when substituted in the given expression makes the two sides identically equal.

An equation which involves the unknown quantity to the first power or degree only is called a **simple equation**; if it contains the square of the unknown quantity it is called a **quadratic equation**; if the cube of the unknown quantity, a **cubic equation**. Thus, the degree of an expression is the power of the highest term contained in it

If an equality involving only an algebraic operation exists between two quantities the expression is called an **identity**, thus $(x+y)^2=x^2+2xy+y^2$ is an identity.

In the equation $2x+7=9$, x represents an unknown number such that twice that number increased by 7 is equal to 9. It is of course obvious that $x=1$, but we may with advantage use this simple example to explain the operation of solving an equation. Before doing so it is necessary to note that as an equation consists of two equal members or sides, one on the left, the other on the right hand side of the sign of equality, the results will still be equal when both sides of the equation are:

(*a*) *equally increased or diminished*, which is the same in effect, as taking any quantity from one side of an equation and placing it on the other side, with a contrary sign;

(*b*) equally multiplied, or equally divided;

(*c*) when each side is raised to the same power, or the same root of each side of the equation is extracted;

(*d*) The signs of all the terms in the equated expressions are changed from + to −, both sides of the equation being altered similarly.

Thus, in the equation $2x+7=9$, subtracting 7 from each side

we get $$2x+7-7=9-7,$$

or $$2x=2.$$

Dividing by 2, then $$x=1.$$

Ex. 1. Solve $4x+2=50$.

Subtracting 2 from each side we get

$$4x=48;$$

$$x=\frac{48}{4}=12.$$

In the example $4x+2=30$ the statement means that if 2 be added to four times an unknown number x, the result will equal 30. By a process of trial, substituting the numbers 1, 2, 3 in turn for x, it will be found that the equation is true only when $x=7$. Then as $4\times 7=28$, this value of x makes the expressions on the left and right-hand sides of the sign of equality numerically equal, or the equation is said to be satisfied.

Instead of subtracting we can **transpose** the 2 in the preceding example from one side of the equation to the other by changing its sign; thus $4x=30-2=28$.

Ex. 2. Solve $4x+5=3x+8$.

Subtract $3x$ from both sides of the equation and we get

$$4x-3x+5=8;$$

next subtract 5 from each side;

$$\therefore x=8-5=3.$$

It is obvious that $+3x$ and $+5$ on the right- and left-hand sides of the equation respectively may be removed from one side to the other (or *transposed*) and appear on the opposite side with changed sign.

Hence the rule for the solution of equations is: *Transpose all the unknown quantities to one side, and all the known quantities to the other; simplify if necessary, and divide by the coefficient of the unknown quantity.*

The rules referred to above, under (*a*), (*b*), (*c*), (*d*), can perhaps be best illustrated by the consideration of a few simple examples.

RULE (*a*).

Ex. 3. Solve $4x+2=3x+4$.

Transposing, $$4x-3x=4-2;$$

$$\therefore x=2.$$

Ex. 4. Solve $5(x+1)=3(x-5)+2$, or $5x+5=3x-15+2$.

Transposing, $$5x-3x=2-15-5;$$

$$\therefore 2x=-18;$$

or $$x=-9.$$

RULES (*b*) AND (*d*).

Ex. 5. Solve $\frac{x}{2}+\frac{x}{3}=x-7.$

Multiplying both sides by 6,

$$3x+2x=6x-42,$$

or

$$5x-6x=-42;$$

$$\therefore\quad -x=-42,$$

or

$$x=42.$$

Ex. 6. Solve $\frac{x}{5}+\frac{16}{5}=\frac{2x}{3}-1.$

L.C.M. of denominators is 15. Hence multiplying both sides by 15,

$$3x+48=10x-15,$$

$$3x-10x=-15-48,$$

$$-7x=-63;$$

$$x=9.$$

EXERCISES. XXVIII.

Solve the following equations

1. $5x-15=2x+6.$
2. $40-6x-16=120-14x.$
3. $3x^2-10x=8x+x^2.$
4. $6ax^3-12abx^2=3ax^3+6ax^2.$
5. $\frac{x}{2}+\frac{x}{3}=13-\frac{x}{4}.$
6. $\frac{x}{2}+\frac{x}{5}=7.$
7. $\frac{2(x-3)}{7}-\frac{5(x+5)}{8}+5\frac{1}{28}=0.$
8. $\frac{2x}{3}+\frac{x}{5}=13.$
9. $\frac{x+3}{x-1}+\frac{x-4}{x-6}=2.$
10. $\frac{2x}{5}+\frac{x}{2}=18.$
11. $\frac{3}{5}(2x-7)-\frac{2}{3}(x-8)=\frac{4x+1}{15}+4.$
12. $\frac{4}{7}(2x-17)-\frac{3}{4}(16-x)=\frac{1}{14}(x+2).$
13. $\frac{x+1}{2}+\frac{x+2}{3}=\frac{5-x}{4}+14.$
14. $\frac{2x+1}{29}-\frac{49-3x}{7}=2-\frac{5x+2}{36}.$
15. $\frac{2x+1}{3}+\frac{4x+2}{5}+\frac{1}{7}=2\frac{12}{35}.$
16. $\frac{x+2}{x-3}+\frac{x-2}{x-6}=2.$
17. $\frac{2x-5}{3}+x=\frac{3x-2}{5}+3.$
18. $\frac{x}{3}-\frac{x-1}{2\frac{1}{2}}=\frac{3x-4}{15}+\frac{x}{12}.$
19. $\frac{2x-1}{5}+\frac{5x+3}{17}=3-\frac{4x-118}{11}.$
20. $\frac{x-\frac{1}{2}}{3}-\frac{\frac{x}{3}-12}{5}=\frac{2x-1}{3}+\frac{9\frac{1}{2}-x}{21}.$

Solve the following equations:

21. $\frac{8x-7}{4}+\frac{3x+7}{3x}+\frac{1}{2}=\frac{6x+1}{3}$. 22. $\frac{x+2}{x-4}+\frac{x+4}{x-2}=2$.

23. $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-4}{x-5}-\frac{x-5}{x-6}$.

24. $\frac{1}{x+2}+\frac{1}{x+3}-\frac{3(x+6)}{(x+2)(x+3)}=6-\frac{6x+23}{x+3}$.

25. $\frac{x}{x+2}-\frac{3x-10}{x+2}=\frac{2x}{3}-x$.

26. $\frac{3x+5}{13}-\frac{5x-2}{11}=\frac{4x-3}{5}-\frac{5x+1}{6}$.

27. In the equation $s=Vt+\frac{1}{2}ft^2$.
  (i) Given $s=10320$, $V=22$, $t=60$; find f.
  (ii) $s=90$, $t=\frac{1}{2}$, $f=32$; find V.

28. In the formula $v^2=2fs$. Given $s=100$, $f=\frac{16}{5}$; find v.

29. In the formula $F=\frac{mv^2}{gr}$. Given $m=12$, $r=3$, $v=40$, $g=32$: find F.

30. Given $\frac{1}{2}mv^2=Fs$. (i) When $m=10$, $F=121$, $s=5$; find v.
  (ii) $m=15$, $v=12$, and $F=30$; find s.
  (iii) If $m=10$, $v=24$, and $s=720$; find F.

31. Find the value of w from the equation
$$\frac{w}{w-390}=\frac{15}{2}.$$

32. In the equation $v^2=u^2+2as$, find the value of a when $v=50$, $u=10$, and $s=100$.

33. In the equation $v^2=V^2+2fs$. Given $V=4$, $f=2\frac{2}{3}$ and $s=33\frac{1}{3}$; find v.

34. If in the equation $\frac{1}{2}mv^2-\frac{1}{2}mV^2=Fs$, $m=12$, $v=11$, $V=5$, and $s=96$; find F.

Problems producing Simple Equations with one unknown Quantity. Having ascertained how to solve an equation involving one unknown quantity, it is only necessary to express the conditions of any given question by means of symbols, and thus to produce equations relating to number, quantity, shape, etc.

Ability to solve an equation readily and easily will be found of the greatest value in all, or nearly all, questions in applied science. The best practice at the outset is to make up simple equations, and having done so, to proceed to solve them. Thus,

as a simple suggestive example, take any number, say 3. Then 4 added to twice 3 gives as a sum 10. Expressing this in symbols we have

$$2x+4=10,$$

from which $x=3$.

Again using the same number we can say that if 6 be added to the number twice the sum will be 18. Expressing this in symbols we have

$$2(x+6)=18,$$

$$2x+12=18,$$

or $x=3$.

A few questions and their solutions are given to show the method by which some required or unknown quantity, x, is obtained from an equation.

Ex. 1. The difference of two numbers is 14. If 9 times the less be subtracted from 6 times the greater, the remainder is 33. What are the numbers?

Let x denote the smaller number.

Then $x+14$ will represent the greater number.

Further, $9x$ is 9 times the smaller number, and $6(x+14)$ is 6 times the greater number.

But when 9 times the smaller is subtracted from 6 times the greater, the remainder is 33.

$$6(x+14)-9x=33,$$

or $6x+84-9x=33$;

$$-3x=33-84=-51,$$

or $x=\frac{-51}{-3}=17$ is the smaller number,

and $x+14=31$ is the greater number.

Ex. 2. What number is that to which, if 8 be added, twice the sum will be 28?

Let x denote the number.

Then $x+8$ is the number with 8 added, and $2(x+8)$ is twice the sum.

But we are told that twice this sum is 28.

Hence, $2(x+8)=28$;

$$2x+16=28.$$

Transposing, $2x=12$,

$$\therefore x=6.$$

Ex. 3. The age of the eldest of three children is equal to the sum of the ages of the other two, the ages of these two being in the ratio of two to three. In ten years the age of the eldest will be five years more than half the sum of the ages of the other two. Find their present ages.

Let $3x$ and $2x$ be the present ages of the two younger children.

Then $5x$ is the present age of the eldest.

In ten years the age of the eldest will be $5x+10$.

$3x+10$, and $2x+10$ the ages of the two younger children.

But $$5x+10=\tfrac{1}{2}(2x+3x+20)+5,$$
$$\therefore\ 5x+10=\tfrac{1}{2}(5x+20)+5,$$
$$\therefore\ \tfrac{5}{2}x=5,\ \text{ or }\ x=2.$$

Hence the ages are 10, 6, and 4.

Ex. 4. The diameters of two pulleys are as 2 to 3, and the sum of the diameters is 30 inches. Find the diameter of each.

Let $2x$ and $3x$ denote the diameters.

Then $$2x+3x=30,\quad \therefore\ x=6.$$

Hence the diameters are 12 inches and 18 inches.

Ex. 5. A stone let fall from the top of a well is found to take 3 seconds to reach the bottom; find the depth of the well.

Here (see p. 84) $$s=\tfrac{1}{2}\times 32\cdot2\times 3^2$$
$$=16\cdot1\times 9$$
$$=144\cdot9 \text{ feet.}$$

As the space described by the falling stone in 3 seconds is 144·9 feet, this also denotes the depth of the well.

Ex. 6. In what time will a stone fall through a distance of 64·4 feet?

Here $$64\cdot4=\tfrac{1}{2}\times 32\cdot2\times t^2;$$
$$t^2=\frac{64\cdot4}{16\cdot1}=4;$$
$$\therefore\ t=2.$$

The time required will be 2 seconds.

EXERCISES. XXIX.

1. Divide a line, 15 inches long, into two parts, such that one is three-fourths the other.

2. A post is one-fourth its length in mud, one-third in the water, and 10 feet above the water; what is its whole length?

3. A man can walk a certain distance in four hours. If he were to increase his rate by one-fifteenth he could walk one mile more in that time; what is his rate?

4. If 10 be added to a certain number, three-fifths the sum is 66; find the number.

5. Two numbers have a difference of 15, and a sum of 59; find the numbers.

6. What number is that to which if 20 be added the sum is equal to three times the required number?

7. If 6 be added to a certain number, twice the sum is 24; what is the number?

8. The difference of two numbers is 6, but if 12 be added to 4 times their sum the whole is equal to 60; find the numbers.

9. A rectangle is 6 feet long, if it were 2 feet wider its area would be 48 feet, find its width.

10. A general after losing a battle found that he had only $\frac{2}{3}$ of his army fit for action, $\frac{1}{6}$ were wounded, and the remainder, 2000 men, were either killed or missing: of how many men did his army consist at first?

11. A roll of cloth was bought at 5s. 6d. a yard, and another roll, 25 yards longer, at 5s. a yard; the two together cost £100. 15s. How many yards were there in each roll?

12. Divide £1120 between *A* and *B*, so that for every half-crown which *A* receives *B* may receive a shilling.

13. The greater of two numbers is 7 times the less, and their difference is 36. Find the numbers.

14. Divide £1000 between two persons so that one may have £10 more than half of what the other has.

15. Find the number so that, if 5 be added to it, the sum is equal to half the excess of 100 over the number.

16. Find a number such that, when diminished by 3, one fourth the remainder may be greater by 2 than one-fifth the original number.

17. Divide 279 into two parts such that one-third the first part is less by 15 than one fifth of the second part.

18. What is the price of an egg when a rise of 20 per cent. in the price would make a difference of 48 in the number which could be bought for 30 shillings?

19. Find two numbers whose sum is 39 and whose difference equals a third part of the greater.

20. *A* has £3 and *B* has £2. 8s. How much shall *A* give *B* that *B* may then have three times as much as *A*?

21. Find two numbers such that their sum is 58, and that half of one exceeds one sixth of the other by 15.

22. A man's age is 40 years, and that of his son 9 years; what will be the age of the father when he is twice as old as the son?

23. A person has a certain sum of money of which he spends [illegible], and then [illegible] of the remainder; if he has 5s. 1d. left, how much had he at first?

24. A man does twice as much work as his son. How long would they take to do a piece of work together which would take the son alone ta days?

25. Find the number of feet traversed in a seconds by a train travelling at a uniform rate of b miles per hour.

26. Divide 17·24 into two parts, such that one quarter of the first added to one third of the second makes 5·06.

27. Divide 12·56 into two parts, one of which is $3\frac{1}{3}$ times the other.

28. A distance of 1·321 feet is to be divided into two parts whose difference is 0·265 feet. What is the length of each part?

29. Divide 20 into two parts, one of which divided by the other is 3·56.

30. An engineer has 50 workmen to whom he pays £16 per day. He pays 6s. 8d. each per day to some of them, and 6s. to the remainder. How many has he at each rate?

31. The ages of a man and his wife added together amount to 72·36 years; fifteen years ago the man's age was 2·3 times that of his wife; what are their ages now?

32. (*a*) Find three consecutive *even* numbers such that their sum is 60.

(*b*) Five years ago A was four times as old as B. At the present time he is three times as old. What is A's age now? [U.E.I.]

33. Two locks and three keys cost 15s. 6d. Three locks and two keys cost 19s. 6d. Find the cost of one lock and of one key. [N.U.T.]

34. A slow train takes c hours and a fast train $(c-1)$ hours to go from A to B. If their average speeds differ by $(c+1)$ miles per hour, find in terms of c the distance from A to B. Find the value when $c=7$. [N.U.T.]

35. The weekly wages for 32 men and 9 boys in a certain workshop is £73. 10s. One man and one boy together receive £2. 16s. What is the weekly wage of each man and boy if all the men are paid the same wage and all the boys receive the same amount? [N.U.T.]

Summary.

Simple Equations.—A statement of equality between two algebraic expressions is called an equation, and it is called a simple equation when it involves only the first power of the unknown quantity.

In an equation the results are still equal if corresponding operations (addition, subtraction, multiplication, division, involution, and evolution) are performed on both sides of the equation.

CHAPTER X.

UNITS OF LENGTH. MEASUREMENT OF LENGTH.

Measurement.—The measurement of a quantity is known when we have obtained a number which indicates its magnitude.

It is necessary, therefore, to select some definite quantity of the same kind, as a unit, and then to proceed to find how many times the unit is contained in the quantity to be measured. The number of times that the unit is contained in the given quantity is the numerical value of the quantity.

Nearly all the quantities with which the man of science and the practical man are called upon to deal are concerned either directly or indirectly with **length, mass,** or **time.**

Fundamental and Derived Units. The primary or fundamental units are three in number, namely *Unit of length, unit of mass, unit of time.*

All derived units are either multiples or sub-multiples of these primary units.

Units of Length.—In order that length may be measured there must be both *a unit* and *a standard.* The unit is a certain definite distance with which all other distances can be compared; and a standard is a bar on which the unit is clearly, accurately, and permanently marked. The two units most generally adopted are the **yard** and the **metre.**

The British System.—In this system the *unit of length* is the *yard.* It may be defined as the distance between two lines on a particular bronze bar when the bar is at a certain temperature (62° F.). The bar is deposited at the Standards Office of the Board of Trade.

The importance of specifying the temperature at which the distance between the two lines on the bar is the exact length will be evident when the alteration in the size of bodies due to a change of temperature is considered.

Sub-Multiples and Multiples of Yard.—The yard is for many purposes inconvenient, and smaller units, or as they are called *sub-multiples* of the unit of length, are often used. To obtain these smaller units, the yard is divided, first into three equal parts each called a *foot*. The foot is again divided into twelve equal parts called *inches*; an inch is further sub-divided into eight, ten, sixteen (or more) equal parts.

Where fractional parts of a foot are required it is often convenient to express feet and the parts of a foot in a simpler form than as feet and inches. This is done by dividing the foot into ten equal parts; the fractional parts are then denoted by $\frac{1}{10}$, $\frac{2}{10}$, etc., or ·1, ·2, etc., of a foot.

Multiples of the yard are used when comparatively long distances have to be expressed. Thus, 1 mile = 1760 yards = 5280 feet. The multiples and sub-multiples of the unit are given in the following table.

It must be at once remarked that in the British System there is no simple relation connecting the unit of length with the units of area, volume, and mass. It is only by a long and troublesome process of multiplication and division—such as reducing feet or miles to inches, ounces to pounds, etc.—that we can proceed to find areas, volumes, etc.

British Measures of Length.

[The unit is divided by 3 and 36, etc.; also multiplied by 2, $5\frac{1}{2}$, 220, and 1760.]

12 inches = 1 foot.	40 poles, or 220 yards = 1 furlong.
3 feet = 1 **yard** (unit).	8 furlongs, 1760 yards, or 5280 feet = 1 mile.
2 yards = 1 fathom.	
$5\frac{1}{2}$ yards = 1 rod, or pole.	

The above table will serve to show how very inconvenient the British Unit and its sub-multiples and multiples are for most purposes, since a cumbersome calculation is required to convert the one into the other.

Fig. 6 shows one edge of a 12-inch *steel scale* or straight edge. The first inch is represented divided into 16 equal parts, and for the convenience of the person using the scale some of the lines are made longer than others. This enables a dimension to be read off much more quickly and accurately than would otherwise be the case. Thus, the cross-line at *c* dividing an inch into two equal parts is seen to be longer than any cross line between *a* and *c*, or between *c* and *b*. In a similar manner the cross lines indicating the quarter inches are longer than those indicating the eighths, these in turn being longer than the sixteenths. As the number of divisions increase the lines naturally become more crowded together, until the distances between the divisions when an inch is divided into 32 equal parts, or those indicating the sixty-fourths, become very minute.

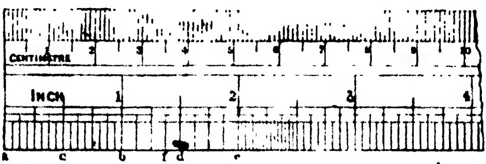

FIG. 6.—Inches divided into 8 and 16 equal parts. A decimetre divided into 10 centimetres and 100 millimetres. (The inches and centimetres are not drawn to a true scale, but their comparative dimensions may be seen.)

The French or Metric System. The Metric System is extensively used for all scientific, and in many cases for commercial purposes, and is in every way better and simpler than the British method.

In the metric system the standard of length is the **metre,** defined originally to be the 10,000,000th part of the length of a north and south line, or meridian, stretching from the equator to one of the poles. The determination of this length was found to be slightly incorrect, and the metre, as in the case of the British standard yard, is best defined as the distance at 0° C. between two marks on a particular platinum bar preserved at Paris, and known as the Mètre des Archives. Copies of the accurate standard metre are to be found in several national repositories in Europe.

Multiples and Sub-Multiples of Metre.—The metre is divided into 10 equal parts called *decimetres*; the decimetre is divided into 10 equal parts each called a *centimetre*: hence a centimetre is one hundredth of a metre, and this sub-multiple of the unit is the most commonly used of the metric measures of length.

The centimetre is divided into 10 equal parts each known as a *millimetre*.

The metre is equal in length to 39·37 inches, and is thus slightly longer than our yard. Its length is roughly 3 feet 3⅜ inches, which number can be easily remembered as it consists throughout of threes.

The foot is equal in length to 30·48 centimetres.

Referring to the upper part of the scale in Fig. 6, the division of a decimetre into 10 centimetres (cm.) is represented, but not to a true scale. Each centimetre is further subdivided into 10 equal parts, each of which is a millimetre.

It will be seen that a length of 10 cm. is approximately equal to 4 inches. A more accurate relation to remember is that a length of 25·4 centimetres or 254 millimetres is equal to the length of 10 inches. Thus, the distance from *a* to *b* may be expressed as 1 inch, 2·54 centimetres, or 25·4 millimetres. The sub-multiples and multiples of the metre are given in the following table.

Metric Measures of Length.

10 **millimetres**	= 1 **centimetre.**
10 **centimetres**	= 1 **decimetre.**
10 **decimetres**	= 1 metre.
10 metres	= 1 **Dekametre.**
10 **Dekametres** or 100 metres	= 1 **Hektometre.**
10 **Hektometres** or 1000 metres	= 1 **Kilometre.**

The Latin prefixes *deci-*, *centi-*, *milli-*, are always used for the tenth, hundredth, and thousandth parts of the unit; in a similar manner the Greek prefixes *Deka-*, *Hekto-*, and *Kilo-*, are used for the multiples of the unit.

It is convenient to remember that, approximately, 2·5 cm. = 1 inch, and 8 kilometres = 5 miles, for these numbers enable a measure to be readily converted from one system to the other.

For accurate calculations, the following relations between the two systems of measurement should be used.

Conversion of British to Metric Measures of Length.

1 inch	2·54	centimetres.
1 foot	30·48	centimetres.
1 yard	0·914	metre.
1 mile	1609·33	metres

Metric to British.

1 millimetre	·039 inch
1 centimetre	·394 inch
1 metre	39·371 inches 3·28 feet 1·094 yards
1 Kilometre	0·621 mile

Abbreviations.—The following abbreviations are generally used, and should be carefully remembered; this may be easily effected by taking the precaution to use the abbreviations on all possible occasions.

Length

in.	is used to	denote	inch or inches
ft.	"	"	feet
dcm.	"	"	decimetre or decimetres.
cm.	"	"	centimetre or centimetres.
mm.	"	"	millimetre or millimetres.
gm.	"	"	gram or grams

Feet and inches are also indicated by the uses of dashes, ′ and ″, at the top and right-hand side of a figure.

Thus, a dimension of 2 feet and 4 inches may be written as 2 ft. 4 in., or 2′ 4″.

This method, though widely adopted in practical work, may (unless care is exercised) sometimes, however, be confusing, for the signs ′ and ″ are also used to designate certain angular measurements; they are also used to designate *primes* and *seconds* in duodecimals.

Measurement of Length.—By means of a rule or scale, as in Fig. 6, any direct measurement can be roughly estimated.

To enable a dimension to be made with a higher degree of accuracy, other and more accurate instruments are required.

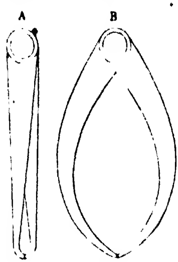

FIG. 7. Inside and outside Calipers

These are known as calipers, screw-gauges, verniers, etc. It is only possible in these pages to refer to a few instruments in common use, and it is far better to see and use these than to read about them.

Calipers.—In many cases it is difficult, if not impossible, to determine a dimension by direct measurement, therefore calipers, a screw-gauge, or other form of instrument must be used for the purpose.

Thus, in measuring the diameter of a sphere or a cylinder, it will be obvious that a scale, like that in Fig. 6, cannot be used for the purpose.

Two common forms of calipers in general use are known as

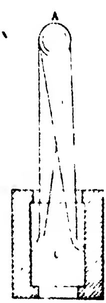

FIG. 8.—Inside Calipers used to determine the width, or diameter, of a cavity.

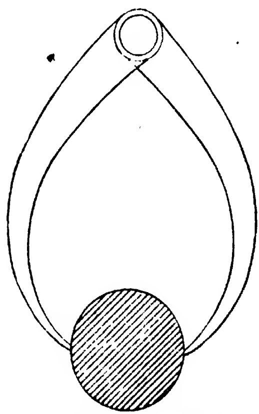

FIG. 9.—Outside Calipers employed to find the diameter of a sphere or cylinder.

inside and *outside calipers*. The former is shown at *A*, and the latter at *B* in Fig. 7. It will probably be obvious that both the forms shown may be used advantageously for many purposes where a straight scale would be altogether unsuitable. One such case is shown in Fig 8, in which a pair of inside calipers is used to ascertain the width, or diameter, of a cavity. Similarly in Fig 9 a pair of outside calipers is used to determine the

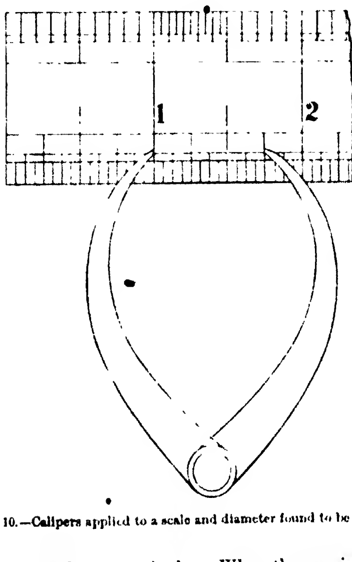

FIG. 10.—Calipers applied to a scale and diameter found to be $\frac{3}{4}$ inch.

diameter of a sphere, or cylinder. When the requisite dimension is obtained, its magnitude in each case can be ascertained by applying the calipers to a scale, Fig. 10, and noticing the length on the scale included between the two edges of the calipers which touched the object measured.

A more accurate method is, however, furnished by what is

called a *screw-gauge*. These instruments are made in several forms, but that in common use is shown in Fig. 11. The instrument consists of a fixed frame F, terminating in two limbs A and B. Through one limb B a fixed screw P passes, and through the other, A, a movable screw S, which is actuated by a spindle terminating in a milled head H. Thus, the rotation of H in a forward or backward direction causes the end Q to approach, or to recede, from the fixed end P. It also causes the sleeve S_1 to which H is fastened, to rotate and also to slide along the fixed part carrying a scale. When the ends Q and P are in contact, the scale on C is covered, and the graduated rim, S_2 is at zero. By rotation of H, Q is made to recede from P, and the distance between the ends, and therefore the diameter, or length, of any object which will just pass between them is given by the reading on the scale on C, together with that on the graduated edge S_2. In this manner, readings can readily be obtained to within $\frac{1}{1000}$ inch or $\frac{1}{100}$ of a millimetre.

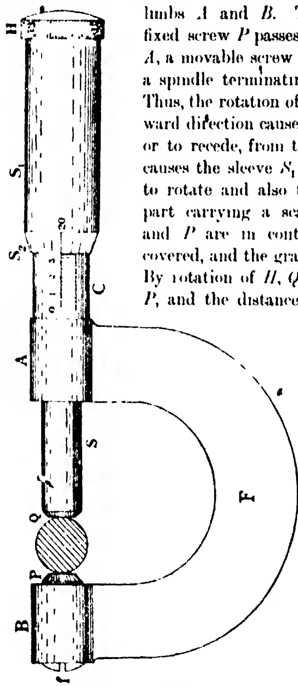

FIG. 11.—Screw Gauge.

Wire Gauge.—To indicate easily the diameter of a wire, or the thickness of a thin plate, is a rather troublesome operation to a practical man. Thus, wire $\frac{1}{16}$ in. diameter is expressed decimally ·0625 in. To avoid the inconvenience of dealing with three or more decimal places, wires are made of standard sizes, and each

size is designated by a particular number. To find the size of a

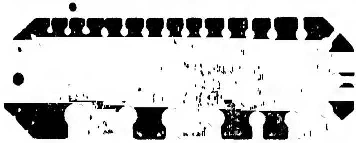

FIG. 12.—Rectangular Wire Gauge

wire, or its standard number, what is known as a wire-gauge is used. This usually consists either of a thin rectangular steel plate (Fig. 12) or a thin circular steel plate (Fig. 13). In each case, to the openings which occur all round the edge, numbers are given, and these numbers indicate the dimensions of wires which just fit the openings. The dimensions corresponding to these numbers are registered in a table.

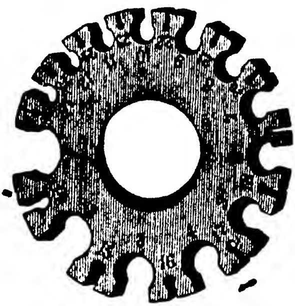

FIG. 13.—Circular Wire Gauge

EXERCISES. XXX.

Find approximately the number of metres in

1. 1 in., 1 ft., 1 yd., 1 pole. **2.** 1 ch., 1 fur., 1 mile.

3. Find the number of poles and chains in 55·3202 metres.

4. Show that 32 metres are approximately equal to 35 yards.

5. Reduce 8 metres, 7 decimetres, 5 centimetres, to yards, feet, and inches.

6. Express 3 miles 7 furlongs 12 poles in metres.

7. A yard being ·9144 of a metre, find the number of metres in $1\frac{1}{4}$ miles.

8. The metre is approximately the ten-millionth part of the distance from the pole to the equator. Taking the earth's circumference as 24855·2 miles, find the length of the metre.

9. Obtain the number of kilometres in 100 miles.

10. If a franc is equal to 9·38 pence, what is the cost in English money of a yard of silk worth $7\frac{1}{2}$ francs a metre.

11. How many steps would a man take in walking $2\frac{1}{2}$ kilometres, each step being 2 feet 8 inches long, and a metre 3 feet 3·37 inches?

12. Obtain (i) the number of feet, and number of metres in ·581250 of a mile; (ii) number of miles, yards and feet in 10,000 metres

13. A vessel steams 16 knots. How many metres per second is this equivalent to? 1 knot = 1·15 miles per hour.

Summary.

Measurement.—A convenient unit is selected, and the number of times the unit is contained in the quantity to be measured is the numerical value of the quantity.

The three quantities to be measured are **Length**, **Mass**, and **Time**.

Primary or fundamental units are comprised in the **unit of length**; the **unit of mass**; and the **unit of time**.

The **derived units** are *multiples, or sub-multiples* of the fundamental units.

The **yard** and the **metre** are the two *fundamental units of length*.

In the **British System**, the *unit of length*, the **yard**, is *the distance between two marks on a certain bronze bar when the bar is at a temperature of* 62° F. The bar is deposited in the Standards Office.

The *derived units* are obtained by dividing the yard into three equal parts, each **1 foot**, which is again subdivided into 12 equal parts, called **inches**; these are again divided into 8, 10, 12, 16, or more equal parts.

Multiples of the unit are 1 mile = 1760 yards, or 5280 feet; $5\frac{1}{2}$ yards = 1 rod; and 220 yards = 1 furlong.

In the French or **Metric System** the *unit of length* is the **Metre**, and is *the distance between two marks on a platinum bar when the temperature of the bar is* 0° C.

The *derived units* are obtained by dividing the metre into 10 equal parts each called a **decimetre**; this is further subdivided into 10 equal parts or **centimetres**, and again into 10, giving **millimetres**.

Measurement of Length.—Instruments used in measurements of length are *Calipers*, *Screw-gauges*, etc.

CHAPTER XI

PLANE ANGLES. ANGULAR MEASUREMENT.

A **plane angle** is the inclination of two lines which meet each other but are not in the same straight line. Thus, if a line AO meet a line OC at O (Fig. 14), the amount of opening between the lines CO, OA is called the angle AOC. If only one angle is formed at O, the angle may be written as the angle O; but if several angles come together at the same point, the middle letter indicates the *vertex* of the angle referred to.

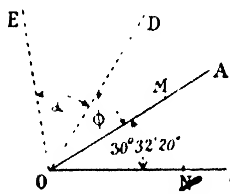

FIG. 14. Plane angles.

It must be very carefully observed that the *angle is independent of the length of the lines forming the sides or legs of the angle.* Thus, the angle O may be accurately described either as the angle AOC, or MON.

Angular Measurement.—In angular measurement, as in linear measure, a suitable *unit* of measurement is selected, and the number of times that any given angle contains the unit is the numerical measure of that angle.

The two units in general use are the **degree** and the **radian.** The degree is obtained by drawing a circle of any convenient radius, and dividing its circumference into 360 equal parts. If two consecutive divisions be joined to the centre, the two lines so drawn contain a length of arc equal to $\frac{1}{360}$th part of the

circumference of the circle, and the angle between them is known as an angle of **one degree.** If, in the circle, two radii are drawn perpendicular to each other, they enclose a quarter of the circle, and hence a right angle consists of 90 degrees, written 90°. Each degree is divided into 60 equal parts, or **minutes**; and each minute is again subdivided into 60 equal parts called **seconds.**

Abbreviations are used for these denominations. Thus, 52° 14′ 20″·5 denotes 52 degrees 14 minutes 20·5 seconds.

The magnitude of an angle in degrees, minutes and seconds may be indicated as shown in the angle COA (Fig. 14), or Greek letters may be used for the same purpose. Thus, the angle COA may be designated by θ, the angle AOD by ϕ, and the angle DOE by α.

Radian.—The remaining unit, the *radian*, may be obtained by drawing as before a circle of any convenient radius, and marking off a portion of the circumference equal in length to the radius. If straight lines be drawn from the extremities of this arc to the centre of the circle, they will enclose an angle of *one radian*, or 57·2958 degrees, approximately 57°·3.

Representation and Measurement of Angles.—As the length of the lines forming the two sides of an angle have no connection with the magnitude of the angle, the actual size is best expressed by the fraction of a circle which the angle in question subtends at its centre. This is done in the following manner.

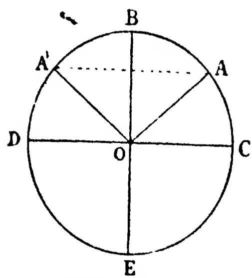

FIG. 15.—Measurement of angles

With centre O and any convenient radius, describe a circle $CBDE$, as shown in Fig. 15.

If we suppose a small pointer (such as the minute hand of a clock or watch) free to move about the centre O, made to coincide with OC and afterwards made to move from C towards B to a position A, through an arc CA one-sixth

of the circumference, then the angle COA is a sixth part of 360°, or is an angle of 60 degrees, written as 60°. In a similar manner, if it moves to B it will trace out an angle of 90 degrees.

When it moves to a position A' it is evident that the angle traced out is greater than a right angle.

All angles greater than a right angle are called *obtuse* angles. Consequently, the angle COA' is an obtuse angle.

Angles less than 90°, or less than a right angle, are *acute*. The angle COA is an acute angle. From the foregoing considerations it will be seen *that an angle is measured by the number of degrees in the arc of the circle, having the vertex of the angle as its centre, intercepted by the two lines forming the angle.*

Comparison of the Magnitudes of Angles.—A comparison of the magnitudes of two angles ABC and DEF (Fig. 16) may be

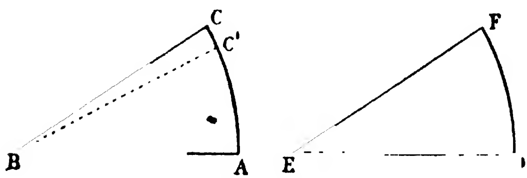

FIG. 16.—Comparison of the magnitudes of two angles.

made by placing the angle DEF on the angle ABC, so that the point E exactly falls upon the point B, and the line DE along the line AB. Then, if the line EF falls on the line BC the angles are said to be equal. The angle DEF is less than the angle ABC if EF falls within BC, as shown by the dotted line BC'. It is larger if it falls outside BC.

This method of *superposition* is readily performed in the following way: Draw from centres B and E two equal arcs AC and DF, so that DE and EF in the one case are equal to AB and BC, respectively, in the other. If the point A be joined to the point C, and D to F, then, if AC is equal to DF, the angles ABC and DEF are obviously equal. Or, using a piece of tracing paper, make a tracing of DEF, and by placing the tracing on ABC, the comparison is readily made.

To set out a given angle.—By means of what is called a 60° set square shown at (i) (Fig. 17), angles of 60°, 30°, 90° and 120° can be set out. Also by means of the 45° set square shown at Fig. 17 (ii), angles of 45°, 90°, and 135° can be marked off. Other angles, viz. 15°, $22\frac{1}{2}$°, and 75° can also be obtained by using these set squares and a pair of compasses.

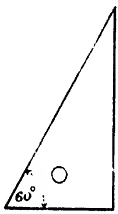

FIG. 17 (i).—A 60° set square

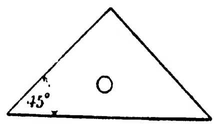

FIG. 17 (ii).—A 45° set square.

To set out an angle of 15°.—Make an angle ABC (Fig. 18) equal to 30° with the set square (i). Bisect the angle ABC by a line BD, then ABD and DBC are each 15°. To bisect the angle ABC we proceed as follows:—With B as centre, and any

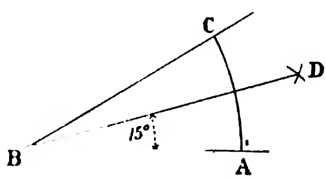

FIG. 18.—To set out an angle of 15°.

convenient radius BA, describe an arc AC. With A as centre, and any radius, describe an arc; and from C, using the same radius, describe another arc cutting the former in point D. Join D to B, then BD bisects the angle ABC, and therefore makes an angle of 15° with AB.

An angle of 22½° is obtained in a similar manner.* Make an angle ABC (Fig. 19) equal to 45° with the set square (ii). Bisect the angle ABC by the line BD as described above; then the angle ABD is equal to 22½°.

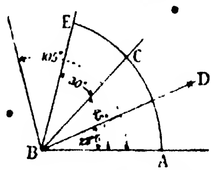

FIG. 19.—To set out an angle of 22½°.

An angle of 75°. To obtain an angle of 75° it is only necessary to use the two set squares. Thus the angle ABC is made equal to 45° by using set square (ii), next adding to this the angle 30° by means of set square (i). The angle ABE = 75°.

If the 60° angle of set square (i) be added to the angle ABC (Fig. 19) the angle obtained will be 105°.

Use of Protractor. Angles which are not conveniently obtained by construction are set out by means of a **protractor.** Two forms of such protractors are shown in Fig. 20. The first

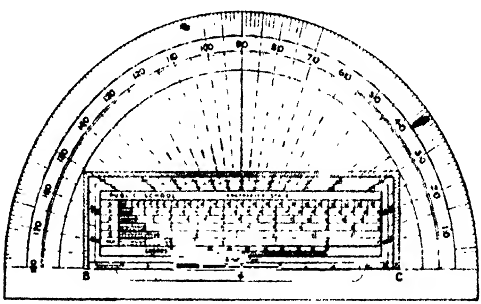
FIG. 20.—Two forms of protractors.

consists of a thin flat rule or scale made of boxwood, ivory, or other material, along the edge of which angles are marked. These marks are obtained from the corresponding division on a semicircle as shown in the illustration.

Ex. 1. Set out by a protractor an angle of 50°.

At the point P (Fig. 21) we place the mark * of Fig. 20 coincident with P, and the edge of the protractor BC with the line PM.

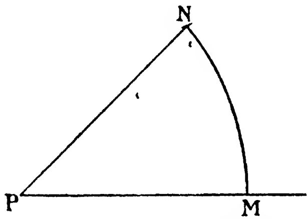

FIG. 21.—To set out an angle by means of a protractor.

Make a mark opposite the division indicating 50° on the protractor. Remove the protractor and join the mark to P. An angle MPN containing 50° will have been made with the given line PM.

How to Use a Protractor to Measure an Angle.—In a similar manner, when used to measure a given angle, the edge BC of the protractor is placed coincident with one of the lines forming the angle. The mark * on the protractor is made to coincide with the vertex of the angle, and the point where the other line crosses the divisions on the scale is noted; this shows, in degrees, the angle required.

Scale of Chords.—At the lower part of a protractor there is nearly always a scale, as shown in Fig. 20, marked "CHORDS." It is known as a scale of chords, and this scale can be used to set out an angle. To do this proceed as follows:—Set a pair of dividers to the space between 0° and 60° on the scale of chords, and, with this distance as radius and P as centre, describe an arc of a circle large enough to contain the required angle. Again apply the dividers to scale of chords and measure a space equal to the number of degrees required. Then, with M as centre and the length so obtained as radius, cut the arc MN at N. Join N to P. Then MPN is the angle required (Fig. 21).

Similarly to measure any given angle ABC (Fig. 22) by means of a scale of chords. Using the compasses, obtain a distance 0° to 60°, and with this distance as radius make an arc DE.

Place one point of the compasses on D and the other coinciding

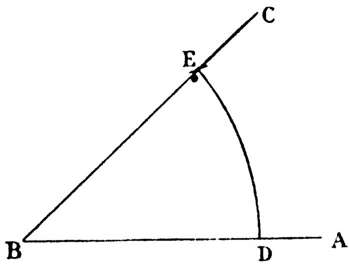

FIG 22.—To measure the magnitude of an angle

with E. Apply the measurement DE to the scale of chords. The reading will give the magnitude of the angle ABC.

Unit of Time.—The sun appears upon the eastern horizon every morning; it rises higher and higher in the sky, and reaches its highest point at noon, when it is seen exactly in a south direction, and therefore shadows thrown by it lie north and south. The interval of time between the instant at which the sun is due south one day and that at which it is due south again the next day is a *solar day*. On account of certain causes which it is not within the province of this book to consider, this interval varies throughout the year. By adding together the lengths of all the solar days in a year, and dividing by 365, an average length of day is obtained. This is called a *mean solar day*. The unit of time used for all scientific purposes is the 86,400th part of the mean solar day, and is a *mean solar second*. Engineers and other practical men use the minute as the unit of time, so that their usual unit is the 1440th of a mean solar day.

Clocks and watches keep mean solar time, but sun-dials keep sun time, or *apparent solar time*, which is sometimes fast and at

other times of the year slow in comparison with mean time. The difference may amount to about fifteen minutes either way.

Summary.

Plane Angle.—A plane angle is the inclination of two lines which meet each other but are not in the same straight line.

Angular Measurement. The two units of angular measurement are the *degree* and the *radian*.

Degree. If any circle be divided into 360 equal parts, and any two consecutive divisions be joined to the centre, the angle enclosed is an angle of 1 degree.

Radian. The radian is the angle at the centre of a circle subtended by an arc equal to the radius. 1 radian $= 57 \cdot 3$ degrees approximately.

Unit of time.—The unit of time, invariably used in scientific work, is one *second*, which is the 86,400th of a mean solar day. The larger unit, one *minute*, is often used by engineers and others in the estimation of power.

CHAPTER XII.

MEASUREMENT OF AREA BRITISH AND METRIC UNITS OF AREA.

Definitions of some Common Plane Figures.—A **triangle** is a plane figure bounded by three straight lines Any one of its three angular points A, B, or C (Fig 23) may be looked upon as the **vertex**; the opposite side is then called the **base** of the triangle. The **altitude** of a triangle is the perpendicular distance of the vertex from the base.

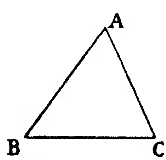

FIG. 23.—A Triangle.

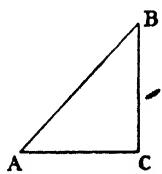

FIG 24.—A Right-angled triangle.

Equilateral Triangle.—When the three sides of a triangle are equal, the triangle is an equilateral triangle; also the angles of the triangle are equal, each being 60°.

Isosceles Triangle.—When two sides of a triangle are equal, the triangle is an isosceles triangle.

A right-angled triangle (Fig. 24) is a triangle one angle (C) of which is a right angle; the side (AB) opposite the right angle is called the **hypotenuse**.

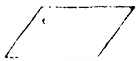

FIG 25.—A Parallelogram

A **parallelogram** is a four-sided figure, the opposite sides of which are parallel.

FIG 26.—A Rectangle

A **rectangle** is a parallelogram having each of its angles a right angle, or, in other words, each side is not only equal in length to the opposite side, but is also perpendicular to the two adjacent sides.

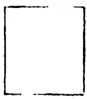

FIG. 27.—A Square

A **square** is a parallelogram which has all its sides equal, and all its angles right angles.

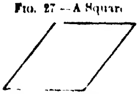

FIG 28.—A Rhombus

A **rhombus** is a parallelogram having all its sides equal, but its angles are not right angles.

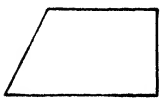

FIG 29.—A Trapezium

A **trapezium** is a four-sided figure which has two of its sides parallel.

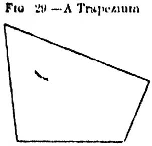

FIG. 30.—A Quadrilateral

A **quadrilateral** is any figure whatever, enclosed by four straight lines.

The **altitude** of a parallelogram is the perpendicular distance between one of the sides assumed as base and the opposite side.

Measurement of Area.—Most readers of this book will probably already know the difference between lengths and areas. But to make quite certain we will take a few simple examples.

Provided with a rule, it would be easy to measure the length of the room and its breadth or width. Now, if we were going to have a carpet put down, we should give the upholsterer the order, and he would pay us a visit to measure the floor. You

know very well it would not be enough for him to measure the length of the room only, or its width only, because both of these are measures of length. To know how much carpet he wants, the workman must find out the amount of surface the floor has, or what is called its **area**. To do this he measures both the length and width of the floor, and when he multiplies them together he gets the area, if the room is a square or rectangular one. If he measures the length and width in feet, then by multiplying them together he gets the area of the floor in square feet; if the measurement of the length and width were taken in inches, the area in square inches would be obtained by multiplying them together.

Whenever areas are measured in this country, square inches, square feet, square miles, or some other unit from square measure is employed. "Square measure" is obtained from "long measure" by multiplication.

Unit of Area. Measurement of area, or square measure, is

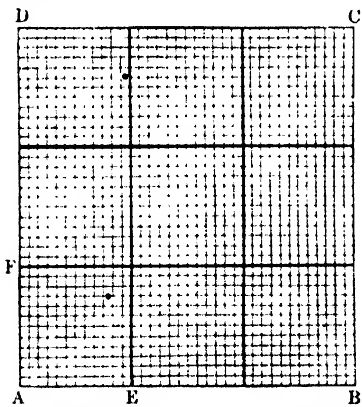

FIG. 31.—1 square yard equals 9 square feet, or 9 × 144 square inches.

then derived from and calculated by means of Measures of

Length. Thus the *unit of area is the area of a square*, the side or edge of which is the unit length.

Area of a Square Yard, or Unit Area.—If the unit length be a yard proceed as follows: Make AB equal to 3 feet, as in Fig. 31, and upon AB construct a square. Divide AB and BC each into 3 equal parts, and draw lines parallel to AB and BC, as in the figure. The unit area is thus seen to consist of 9 smaller squares, every side of which represents a foot; thus the unit area, the square yard, contains 9 square feet. The smaller measures of length, the foot and the inch, are much more generally used than the yard. If the unit of length AE (Fig. 31) be 1 *foot*, the unit of area AEF is 1 square foot. In a similar manner, when the unit of length is 1 inch, the unit of area is 1 square inch. If the unit of length be 1 centimetre, the unit of area is 1 square centimetre (Fig. 32).

If the side of the square on AE (Fig. 31) represent, on some convenient scale, 1 foot, then by dividing AE and AF each into 12 equal parts, the distance between consecutive divisions would denote an inch. If through these points lines be drawn parallel to AE and AF respectively, it will be found that there are 12 rows of squares parallel to AE, and 12 squares in each of these 12 rows. Hence the area of a square foot represents 144 square inches.

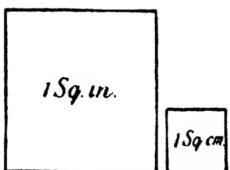

FIG. 32.—Square inch and square centimetre.

British Measures of Area or Surface.

[*Unit area* = 1 *square yard.* Larger and smaller units obtained by multiplying by 4840 and dividing by 9, 27, 1296 and 3888.]

144 square inches	= 1 square foot.
1296 square inches or 9 sq. ft.	= 1 square yard.
4840 square yards	= 1 acre.
640 acres	= 1 square mile.

When comparatively large areas, such as the areas of fields, have to be estimated, the measurements of length, or linear

measurements, are made by using a chain (known as **Gunter's Chain**) 22 yards long. Such a chain is subdivided into 100 links. The square measurements, or areas, are estimated by the *square chain*, or 484 (22 × 22) square yards in area. Or the area of a square, the length of one side of which is 22 yards, is 100 × 100 = 10000 sq. links; for each chain consists of 100 links. Hence we have the relation

1 chain	= 22 yards	= 100 links.
1 square chain	= 484 square yards	= 10000 sq. links.
10 square chains	= 4840 square yards	= 1 acre.

144 square inches (sq. in.)	= 1 square foot (sq. foot)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square perch, rod, or pole (sq. po.)
40 square poles	= 1 rood (r.).
4 roods	= 1 acre (ac.) = 4840 square yards.
640 acres	= 1 square mile (sq. m.).

Metric Measures of Area.—As the metric unit of length is the metre, the unit of area (Fig. 33) is a square *ABDE*, having

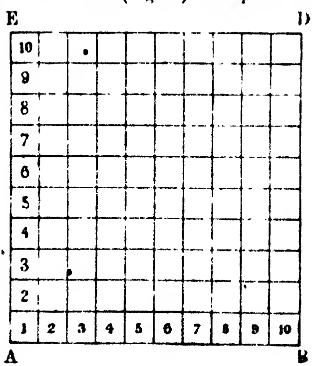

Fig. 33.—Representing a square metre divided into 10 sq. decimetres. Scale $\frac{1}{20}$

the length of its edge equal to 1 metre, and its area consequently equal to 1 square metre.

If AB and BD are each divided into 10 equal parts and lines drawn parallel to AB and BD, as shown, the unit area is divided into 100 equal squares, each of which is a square decimetre.

In scientific work the **centimetre** is the unit of length usually selected, and the unit of area is then one **square centimetre** (Fig. 32).

Metric Measures of Area.

100 square millimetres = 1 square centimetre.
10000 „ „ = 100 sq. cm. = 1 sq. decim.
100 „ decimetres = 1 square metre.

Conversion Table.

British to Metric.		Metric to British.	
1 sq. in.	= 6·451 sq. cm.	1 sq. cm.	= 0·155 sq. in.
1 sq. ft.	= 929 sq. cm.	1 sq. m.	= 10·764 sq. ft.
1 sq. yard	= 8361·13 sq. cm.	1 sq. m.	= 1·196 sq. yard.
1 acre	= 4046·7 sq. metres	1 sq. km.	= 0·3861 sq. mile.
1 sq. mile	= 2·59 sq. km.		
	= $2·59 \times 10^{10}$ sq. cm.		

EXERCISES. XXXI.

1. Find the number of square metres in (i) 10 sq. ft., (ii) 10 sq. yds.
2. Find the number of square metres in a quarter of an acre.
3. Find the number of square metres in 1000 square yards.
4. Express 2 sq. ft. 25 in. as the decimal of a square metre.
5. Reduce 1000 sq. in. to square metres.
6. Find the number of square miles in 25898945 sq. metres.
7. How many square inches are there in 5 ac. 3 r. 35 p. 4 sq. ft. 101 sq. in.?
8. Find which is greater, 10 sq. metres or 12 sq. yds., and express the difference between these areas as a decimal of a square metre.

Summary.

Measurement of area is obtained directly from the measurement of length.

The unit of area is the area of a square upon a line of unit length. As the unit of length in the British system is the yard, the *British unit of area is the* **square yard**. The smaller units—the *square foot* and the *square inch*—are, however, much more convenient, and are generally used.

The *Metric unit of area is the square metre.* The **square centimetre**, or the area of a square having each side 1 centimetre long, is generally used.

CHAPTER XIII.

AREAS OF PLANE FIGURES. MENSURATION OF RECTANGLE AND PARALLELOGRAM. PERIMETER.

Areas of Plane Figures. When the numbers of units of length in two lines at right angles to each other are multiplied together, the product obtained is said to be a quantity of two dimensions, and is referred to as so many square inches, square feet, square centimetres, etc., depending upon the units in which the measures of the lengths are taken. The result of the multiplication gives what is called an **area**. Or, briefly, the area of a surface is the number of square units (square inches, etc.) contained in the surface.

It is obvious that although square inches or units of area are derived from, and calculated by means of, linear measure, those quantities only which are of the same dimensions can be added, subtracted, or equated to each other. Thus, we cannot add or subtract a line and an area. Results obtained in such cases would be meaningless.

It must be observed, too, that the two lengths multiplied together to obtain the area are *perpendicular to each other.* This applies to the calculation of all areas.

Addition and Subtraction of Areas.—To add or subtract graphically the areas of two squares, the sides of which are AB and CD (Fig. 34), *i.e.* to find the side of a square having an area equal to the sum or difference of the areas of the two given squares, we proceed in the following way.

1. **To find the Sum of two Squares.**—Draw two lines at right angles to one another, as BD and BA (Fig. 34). Make

BD equal in length to CD, and BA equal in length to the side of the second larger square.

Join AD.

Then the square constructed on AD will have an area equal to the sum of the areas of the squares constructed on AB and CD.

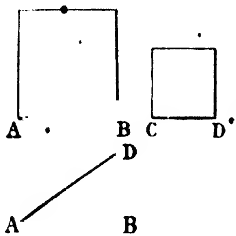

FIG. 34.—Addition and subtraction of areas

[This is an application of a proposition in Euclid (Book I., Prop. 47) which states that *The square on the side of a right-angled triangle opposite to the right angle is equal to the sum of the squares on the other two sides.*

Or $$AD^2 = AB^2 + BD^2.$$
$$AD = \sqrt{AB^2 + BD^2}.]$$

Ex. 1. Let $AB = 4$ and $BD = 3$.

Then $$AD = \sqrt{16+9} = \sqrt{25} = 5.$$

2. **To find the Difference between two Squares.**—Let AB and CD (Fig. 34) be the sides of two given squares. To find the side of a square, the area of which will be equal to the difference of the areas of the two squares.

Draw two lines at right angles to one another, as AB and BD, making BD equal in length to CD, the side of the smaller square.

With D as centre, and a radius DA equal to AB, the side of the larger square, describe an arc of a circle.

Let A be the point of intersection of the arc with the horizontal line BA.

Then BA is the length of the side of the square required.

Ex. 2. Let $AB = 10$ cm. and $DC = 8$ cm.

As before draw two lines at right angles making BD equal to 8 cm. With D as centre, radius 10 cm., describe an arc of a circle cutting AB at A.

Then AB is the length required.

$$AB = \sqrt{10^2 - 8^2} = \sqrt{36} = 6.$$

Area of a Rectangle. - *The number of units of area in a rectangular figure is found by multiplying together the numbers of units of length in two adjacent sides.*

Thus, if AB and BC (Fig. 35) are two adjacent sides of a rectangle; the area of the figure is the product of the number of units of length in AB, by the number of units of length in BC.

The number of units in the lowest line of the figure AB is usually called the *base*, or length; and the units in line BC, perpendicular to this, the *altitude*, or breadth.

Hence area = *base* × *altitude*,

or = length × breadth = ab.

When the adjacent sides are equal in length the figure becomes a square, and as b is now equal to a, the *area of a square* = a^2.

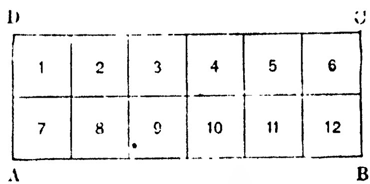

FIG 35 — Area of a rectangle

Ex. 1. If the base AB and height BC are 6 and 2 units of length respectively, the area of the rectangle is 12 square units. If AB be divided into 6 equal parts and BC into 2, then by drawing lines through the points of division parallel to AB and BC, the rectangle is divided into 12 equal squares (Fig. 35).

The area is obtained in a similar manner when the two given numbers denoting the lengths of the sides are not whole numbers.

Ex. Obtain the area of a rectangle when the two adjacent sides are 5 ft. 9 in. and 2 ft. 6 in. in length respectively.

We may either reduce to inches before multiplying:

Thus, 5 ft. 9 in. = 69 inches;

and, 2 ft. 6 in. = 30 inches.

∴ area of rectangle = 69 × 30 square inches

= 2070 square inches = 14·375 square feet.

Or, instead of first reducing the feet to inches and afterwards multiplying, we may proceed as follows:

$$5 \text{ ft. } 9 \text{ in.} = 5\tfrac{3}{4} \text{ feet and } 2 \text{ ft. } 6 \text{ in.} = 2\tfrac{1}{2} \text{ feet.}$$

$$\therefore \text{ area of rectangle} = 5\tfrac{3}{4} \times 2\tfrac{1}{2} \text{ square feet}$$

$$= \tfrac{23}{4} \times \tfrac{5}{2} \text{ square feet} = 14\tfrac{3}{8} \text{ square feet.}$$

It is an easy matter to verify the latter example as shown in Fig. 36, where $AB = 5\frac{3}{4}$ ft. and $BC = 2\frac{1}{2}$ ft.

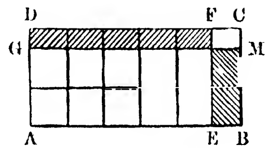

FIG. 36. Area of a rectangle.

The area of the rectangle $AEG = 10$ square feet.

Area of the rectangle $EBM = 2(1 \times \frac{3}{4}) = 1\frac{1}{2}$ square feet.

" the rectangle $DFG = 5(1 \times \frac{1}{2}) = 2\frac{1}{2}$

" rectangle $FCM = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

Hence the whole area of rectangle $ABCD$

$$= 10 + 1\tfrac{1}{2} + 2\tfrac{1}{2} + \tfrac{3}{8} \text{ square feet}$$

$$= 14\tfrac{3}{8} \text{ square feet.}$$

Area of a Parallelogram.—The rectangle is a particular case of the parallelogram, and

area of parallelogram = *base* × *altitude*.

This may also be shown as follows:

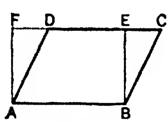

FIG. 37.—Area of a parallelogram.

Let $ABCD$ (Fig. 37) be the given parallelogram.

Draw AF and BE perpendicular to AB, and meeting CD at E, and CD produced at F, then the rectangle $ABFE$ is equal in area to the parallelogram $ABCD$.

Hence, area of parallelogram

= *base* × *altitude* = ab;

or, *the distance between a pair of parallel sides multiplied by one of them.*

If the areas shown in Fig 37 be drawn on a sheet of paper, then by cutting out the right-angled triangle BCE, inverting and fitting it to the other end in the way the figure makes clear, the parallelogram is converted into a rectangle.

When a number of rectangular pieces of millboard, card-board, or thin wood (a pack of cards may be conveniently used), lie each directly upon one another the side edges form a rectangle, the area of which is equal to $AB \times BC$, units of area, as shown by the rectangle $ABCD$ (Fig 38)

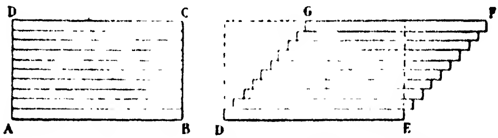

FIG 38. Apparatus to show that when the bases and heights of a rectangle and a parallelogram are the same, the areas are equal.

When these pieces are made to slide over each other, a parallelogram $DEFG$ (Fig. 38) is formed by the side edges, and it is obvious that the area is unaltered by merely changing the position of the pieces, since the base and height remain unaltered.

Ex. 1 If the length of the base AB is 4 ft, and the altitude $1\frac{1}{2}$ ft. in length, find the area.

$$\text{Area} = 4 \times 1\tfrac{1}{2} = 6 \text{ sq ft}$$

Ex. 2. Find the area of a rectangular board of length $12\frac{1}{2}$ ft. and width 1 ft 6 in.

Area $= 12{\cdot}5 \times 1{\cdot}5 = 18{\cdot}75$ sq. ft

Perimeter.—The term *perimeter of a figure* is used to denote the sum of the lengths of all its sides. Thus, the perimeter of a rectangle $ABCD$ (Fig. 39) is the sum of the lengths of its four sides, AB, BC, CD, and DA; but, as the two opposite sides of a rectangle are equal to each other in length, the perimeter is evidently

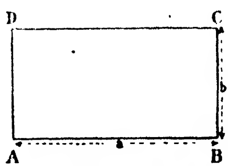

FIG. 39.—Perimeter of a rectangle is $2(a+b)$.

twice the length of one side, together with twice the length of an adjacent side; or,

perimeter $=2(a+b)$ where a and b denote the lengths of adjacent sides.

The *papering*, *painting*, or *distempering* of the walls of a room is usually estimated at the rate of so much per square foot, or per square yard.

Hence, the cost of the work is obtained by multiplying the length and breadth together to obtain the area, and finally multiplying by the unit of cost. The four walls of a room may

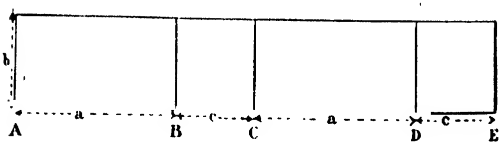

FIG. 40.

be shown as at AB, BC, CD, and DE (Fig. 40), where a represents the length of one wall and b its breadth, or, height, and c and b the corresponding dimensions of an adjacent wall. In many cases the opposite walls are equal in area, and then the area of the four walls is $2(ab+cb)$, or $2b(a+c)$.

The areas of windows, doors, etc., would usually have to be subtracted from the area thus obtained.

The cost of *staining* and *varnishing* floors is also often estimated in a similar manner by obtaining the area in square feet, or square yards, and multiplying the area by a unit of cost.

Ex. 1. Find the cost of painting the two sides of a rectangular wooden partition, length 37 ft., height 5 ft. 3 in., at 2s. per sq. yd.

$$\text{Area of one side} = 37 \times 5\tfrac{1}{4} \text{ sq. ft.}$$

$$\text{,, both sides} = \frac{37 \times 5\frac{1}{4} \times 2}{9} \text{ sq. yds.}$$

$$= 43\cdot166 \text{ sq. yds.}$$

$$\text{Cost} = 43\cdot166 \times 2 \text{ shillings} = £4.\ 6s.\ 4d.$$

Ex. 2. The dimensions of a room are: length 21 ft. 9½ in., width, 15 ft. 7 in., and height 8 ft. 1½ in. Find the length of paper required

for the four walls, and its total cost, knowing that the width of the paper is $22\frac{1}{2}$ inches, and its cost 1s. 3d. per yard.

Area of walls

$$=2(21\text{ ft. }9\tfrac{1}{2}\text{ in.}\times 8\text{ ft. }1\tfrac{1}{2}\text{ in})+2(15\text{ ft }7\text{ in}\times 8\text{ ft. }1\tfrac{1}{2}\text{ in.})$$

$$=74\tfrac{3}{4}\times 8\tfrac{1}{8}\text{ sq. ft}=\frac{74\frac{3}{4}\times 8\frac{1}{8}}{9}\text{ sq. yds.}$$

$$\text{Length of paper}=\frac{74\frac{3}{4}\times 8\frac{1}{8}}{9\times\frac{22\frac{1}{2}}{36}}=107\tfrac{35}{36}\text{ yds.}$$

$$\text{Cost of paper}=107\tfrac{35}{36}\times 1\tfrac{1}{4}\text{s.}=\pounds 6.\ 14\text{s. }11\tfrac{1}{2}\text{d.}$$

Ex. 3. Find the cost of papering the walls and ceiling of a room; the dimensions are: length 36 ft., width 18 ft., and height 12 ft., the cost for the walls being 1s. per sq. yd., and for the ceiling 1s. 6d. per sq. yd.

$$\text{Area of the four walls}=2(36\times 12)+2(18\times 12)\text{ sq. ft.}$$

$$=1296\text{ sq. ft., or }\tfrac{1296}{9}=144\text{ sq. yds.}$$

$$\therefore\ \text{cost of walls}=144\times 1\text{s.}=\pounds 7.\ 4\text{s.}$$

$$\text{Area of ceiling}=\frac{36\times 18}{9}=72\text{ sq. yds.}$$

$$\therefore\ \text{cost of ceiling}=72\times 1\tfrac{1}{2}\text{s.}=\pounds 5.\ 8\text{s. }0\text{d.}$$

$$\text{total cost}=\pounds 12.\ 12\text{s.}$$

Ex. 4. Find the length of wall paper required to paper the four walls of a room, the width of the paper being 1 ft. 9 in. The dimensions of the room are: length 24 ft., width 18 ft., and height 12 ft. There are three doorways 6 ft. high and $3\frac{1}{2}$ ft. wide, and an opening 7 ft. high and 5 ft. wide, which are not to be papered.

$$\text{Area of walls}=2(24\times 12)+2(18\times 12)=1008\text{ sq. ft.}$$

$$\text{Areas to be deducted}=(7\times 5)+3(6\times 3\tfrac{1}{2})=98\text{ sq. ft.}$$

$$\text{Hence area to be papered}=1008-98=910\text{ sq. ft.}$$

$$\therefore\ \text{length of paper required}=\frac{910}{\frac{21}{12}}=520\text{ ft.}$$

Carpeting.—Usually a carpet is made in one long continuous piece of a given width. Hence, to ascertain the length of carpet required to cover the floor of a room, it is necessary to obtain the area of the floor, and this area divided by the width of the carpet will give the length of it required.

Ex. 1. If a carpet is $\frac{3}{4}$ yard wide, find the length required to cover the floor of a room, length 24 ft., width 18 ft.; find also the cost at 5s. per yard.

Let $ABCD$ (Fig. 41) represent a plan of the room, AB being 24 ft., and BC 18 ft.

As 18 ft. $=6$ yds., and $6 \div \frac{3}{4} = 8$, if DA be divided into 8 equal parts at E, F, G, H, K, L, M, each of the portions indicated will be $\frac{3}{4}$ yard, and the length of carpet required will be

$$8 \times 24 = 192 \text{ ft., or } 64 \text{ yards.}$$

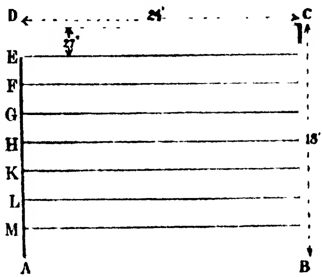

FIG. 41.

The previous result may be obtained in a simpler manner.

The area of the carpet must be equal to the area of the floor.

$$\therefore \text{ Length of carpet required} = \frac{\text{area of floor}}{\text{width of carpet}}.$$

Thus, in the above example,

$$\text{area of floor} = 24 \times 18 \text{ sq. ft.}$$
$$= \frac{24 \times 18}{9} = 48 \text{ sq. yds.}$$

$$\text{Length of carpet required} = 48 \div \tfrac{3}{4} = 64 \text{ yds.}$$
$$\text{Cost of carpet} = 64 \times 5\text{s.} = £16.$$

Ex. 2. The length of a room is 36 ft., and its width 22 ft. A border 2 ft. wide is to be stained all round the room, and the central part covered with a carpet $\frac{3}{4}$ yard wide. If the staining cost 6d. per square foot, and the carpet 5s. per yard, find the total cost.

Let $ABCD$ (Fig. 42) represent a plan of the floor, and the shaded portion, the area, to be stained. This area will be obtained by subtracting the area of the inner rectangle from the area of the outer rectangle. The sides of the inner rectangle are each 4 ft. less than those of the outer.

$$\therefore \text{ area to be stained} = 36 \times 22 - 32 \times 18 = 792 - 576 = 216 \text{ sq. ft.}$$
$$\therefore \text{ cost of staining} = 216 \times \tfrac{1}{2} = 108\text{s.} = £5.\ 8\text{s.}$$

Length of carpet required $= \dfrac{576}{9 \times \frac{3}{4}} = 85\frac{1}{3}$ yds.

Cost of carpet $= 85\frac{1}{3} \times 5$s. $=$ £21. 6s. 8d.

total cost $=$ £21. 6s. 8d. $+$ £5. 8s. $=$ £26. 14s. 8d.

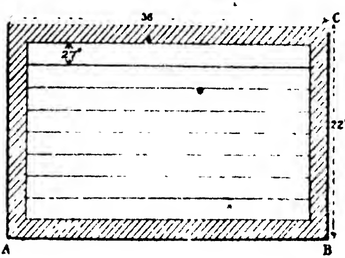

FIG. 42.

Linings.—The cost of lining the inside of a tank or cistern with lead or zinc is generally estimated at so much per square foot. When sheet lead is used for this and other purposes it is usually referred to by its weight. Thus, "5 lbs. lead" refers to sheet lead, and means that a square foot of it weighs 5 lbs. Hence, when the number of square feet in the area which has to be covered is known, the weight of lead required can readily be obtained by multiplication.

Ex. 1. The length of a rectangular box without a lid is 8 ft., width 3 ft., and depth 4 ft. If the surface is to be covered with 6 lbs. lead, find the weight of lead required.

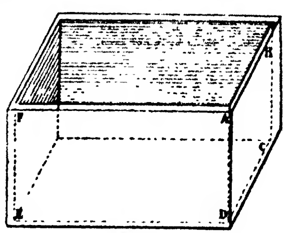

FIG. 43

Area of the end *ABCD* (Fig. 43)

$= 4 \times 3 = 12$ sq. ft.;

$\therefore$ area of both ends

$= 24$ sq. ft.

Area of the side $AFED = 8 \times 4$;

$\therefore$ area of the two sides $= 8 \times 4 \times 2 = 64$ sq. ft.

Area of base $= 8 \times 3 = 24$ sq. ft.

Hence total area to be covered $= 24 + 64 + 24 = 112$ sq. ft.

$\therefore$ weight of lead required $= 112 \times 6 = 672$ lbs.

EXERCISES: XXXII.

1. (i) Calculate the length of the side of a square field which is an acre in area. (ii) What fraction of an acre is a field 55 yards long by 40 yards wide?

2. Find the number of yards in a side of a square field, the area of which is 11 ac. 36 per.

3. A field of 20 ac. is four times as long as it is broad; find the length and breadth in yards.

4. The length of a room is $31\frac{1}{2}$ ft., and the area 46 sq. yds.; what is the breadth?

5. The length and width of a rectangular enclosure are 386 and 300 ft. respectively; find the length of a diagonal.

6. The foot of a ladder is at a distance of 36 ft. from a vertical wall, the top is 48 ft. from the ground; find the length of the ladder.

7. The height of a lean-to roof is 3 ft. 6 in., and the span 12 ft. (Fig. 44); how many square feet of felt will be required to cover 20 ft. length of the roof?

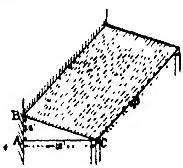

FIG. 44.

8. The side of a square is 24 ft. 6 in.; what is its area?

9. Each side of an equilateral triangle is 10 ft.; find the length of the perpendicular from the base to the vertex.

10. What length of matting, $\frac{3}{4}$ yd. wide, will be required to cover a room 39 ft. 6 in. long by 25 ft. 6 in. wide? What is the cost of the same at 4s. 6d. a yard?

11. A quantity of matting, 37 ft. 9 in. long, and 7 ft. 6 in. wide, will just cover a room; what is the width of matting, $75\frac{1}{2}$ ft. long, which will cover the same room?

12. A plot of ground required for building a warehouse is a square of 90 ft. 9 in. side; find its cost at £1. 12s. per square yard.

13. What would be the cost of painting the four walls of a room whose length is 24 ft. 3 in., breadth 15 ft. 8 in., and height 11 ft. 6 in., at 2s. per square foot?

14. What will be the expense of paving an area which measures 35 ft. 10 in. by 18 ft. 6 in., at 6s. 3d. per square yard?

15. Calculate the cost of the paper for the walls of a room, 20 ft. long, 17 ft. broad, and 12 ft. high, allowing 42 sq. ft. for doors and windows, the price of the paper being 2s. 8d. per piece of 12 yds. long and 21 in. broad.

16. What length of matting, ¾ yd. wide, will cover the floor of a hospital 31 ft. 6 in. long by 18 ft. 9 in. wide? What is the cost of the matting at 2s. 3d. a yard?

17. A rectangular field, 400 yds. long and 396 yds. broad, is let for £54 a year; find the rent of an acre of the same field.

18. How many square feet of glass are required to glaze 5 windows, each containing 14 panes of glass, and each pane measuring 18 in. by 6 in.?

19. How many 3-inch squares can be cut out of a 12 inch square?

20. A gardener has a piece of matting, 73 yds. 1 ft. 8 in. long and 3 ft. 9 in. wide, to cover a wall 94 ft. long and 10 ft. high; how many square feet of wall remain uncovered?

21. What is the cost of carpeting a room, 25 ft. 9 in. long by 22 ft. 6 in. wide, at 5s. 4d. per square yard?

22. Find the cost of papering a room, 18 ft. 1 in. long by 14 ft. 8 in. and 12 ft. 6 in. high, with a paper 30 in. wide, and costing 4½d. per yard.

23. A rectangular field is 60 ft. long by 40 ft. wide; it is surrounded by a road of uniform width, the whole area of which is equal to the area of the field: find the width of the road.

24. Find the rent, at £2. 5s. an acre, of a rectangular park ½ mile long and ¼ mile wide.

25. A rectangular garden contains 1200 sq. yds.; if the length is to the breadth as 4 to 3, what will the fencing cost at 3s. 6d. per yard?

26. The two sides of a rectangular field are found to be ·0876 miles and ·0056 miles, respectively; find the area.

27. The length of a field of 2 acres is 7½ poles; find its width.

28. Find how long it will take to walk round a square field containing 7 ac. 2601 sq. yds., at 3 miles an hour.

29. Find the cost of lining a rectangular cistern with "7-lb. lead," at 3d. per pound; the inside dimensions are, length 3 ft. 2 in., width 2 ft. 8 in., and depth 2 ft. 6 in.

30. A rectangular room is 30 ft. long and 22 ft. wide; the height from the top of the skirting to the cornice is 11 ft. How many yards of paper 21 in. wide will be required to cover the walls, assuming that the door, windows, and other openings, just compensate the waste involved in matching the pattern?

31. A carpet covering the central portion of a room cost £3. 6s. 8d. The room was 16 ft. long by 12 ft. wide, and between edge of carpet and the walls a distance all round of 2 ft.; what was the cost of the carpet per square yard?

32. Find the cost of carpeting a room, 34 ft. 6 in. long by 18 ft. 4 in. wide, at 3s. 9d. a square yard.

33. The cost of carpeting a room, 26 ft. 8 in. long, with carpet at 4s. 6d. a square yard is £9. 10s.; find the width of the room.

34. The diagonal of a square is 3362 ft.; find the length of a side of the square.

35. The area of a rectangular field contains 462 sq. yds., its length is 25 yds. 2 ft.; find its width.

36. Find the cost of putting a fence, at 3s. 6d. per yard, round a square field, whose area is two acres.

37. A garden, 77 ft. 4 in. long by 50 ft. 4 in. wide, is surrounded by a wall 14 in. thick; within the wall, round the four sides, there is a bed 6 ft. wide, then a pathway 5 ft. wide, and grass in the middle: find the areas of the wall, the bed, the pathway, and the grass.

38. If the cost of putting a fence round a square enclosure is £36. 13s. 4d., find the side of the square, and price per yard of the fence, when the area is 10 acres.

39. A square field has an area of 2 acres; what will it cost to enclose at 5s. 6d. per lineal yard of its boundary, and to cut a path diagonally across it at 1s. $6\frac{1}{2}$d. the lineal yard?

40. The dimensions of a room are, length 21 ft., width 16 ft., and height 11 ft. There is a doorway 7 ft. high and 3 ft. wide, and two windows 7 ft. by 4 ft. Find cost of papering the room with paper, 2 ft. wide, at 3d. per yard.

41. A rectangular field contains $2\frac{1}{4}$ acres. If the length to the breadth is 11 : 8, find the perimeter. [U.E.I.]

42. A rectangular room is a yds. long, b ft. wide and c ft. high. Find (a) the area of the walls, (b) cost in shillings of distempering the ceiling at 3d. per sq. yd. (c) Find the numerical values when $a=5$, $b=13$, $c=12$. [N.U.T.]

43. A rectangular field is 130 yds. long and 100 yds. broad. Two paths, each 10 ft. wide, are made across it so as to divide the field into four equal rectangular plots. How much would be realized by selling the plots at £57. 15s. per acre? [L.C.U.]

44. Find the length of the side of a square plot of grass if, when an extra strip of turf one yard in width is put completely around it, the area is increased by 40 sq. yds. [L.C.U.]

Summary.

Area of a Rectangle = *base* × *altitude* = *length* × *breadth*.

Area of Parallelogram = *base* × *altitude*.

Perimeter.—The perimeter of a figure denotes the sum of the lengths of all its sides.

CHAPTER XIV.

MENSURATION OF TRIANGLE, RHOMBUS, TRAPEZIUM, AND QUADRILATERAL.

Area of a Triangle. In Fig. 45 the rectangle $ABCD$ and the parallelogram $ABEF$ on the same base AB and of the same altitude, are equal in area.

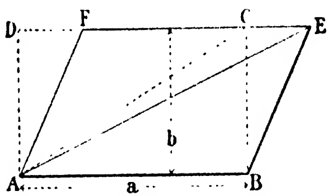

FIG. 45.—Area of a triangle.

When A is joined to C and E it is easily seen that the triangle ACB is half the rectangle $ABCD$, and the triangle AEB is half the parallelogram $ABEF$.

Hence, the two triangles are equal in area, and the area in each case is equal to *half the product of the base and the altitude.*

$$\therefore \text{ area of a triangle} = \tfrac{1}{2}(\text{base} \times \text{altitude}) = \tfrac{1}{2}ab.$$

This result is shown graphically (Fig. 46). The rectangle $ABHL$ on the same, or an equal base, and half the height has the same area as the triangle ABC.

Ex. 1. In a right-angled triangle the three sides are 3, 4, and 5 ft. respectively, find the area.

If the base be 4 then the height is 3.

$$\text{Area} = \tfrac{1}{2} \times 4 \times 3 = 6 \text{ sq. ft.}$$

Ex. 2. If the base of a triangle be $10\tfrac{3}{4}$ yds., and the height $4\tfrac{1}{5}$ yds., what is the area?

$$\text{Area} = \tfrac{1}{2}(10\tfrac{3}{4} \times 4\tfrac{1}{5}) = \tfrac{1}{2}(\tfrac{43}{4} \times \tfrac{21}{5}) = 22.575 \text{ sq. yds.}$$

If a *right-angled triangle is also isosceles*, so that $AB = BC$ (Fig. 46) then denoting AB and BC each by unity,

$$AC = \sqrt{1^2+1^2} = \sqrt{2}.$$

If the equal sides are of unit length, the area is

$$\frac{1 \times 1}{2} = \tfrac{1}{2} \text{ unit of area}$$

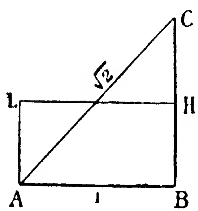

FIG. 46.—Area of an isosceles triangle.

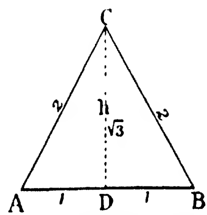

FIG. 47.—Area of an equilateral triangle.

In an *equilateral triangle* the three sides are all of equal length, and each angle is 60°.

If, as in Fig. 47, a line CD be drawn perpendicular to the base, and meeting the base AB at D, then the base is bisected at the point D so that $AD = DB$.

If each of the equal sides be two units in length,

then $\qquad AD = 1,$

and $\qquad CD = \sqrt{2^2 - 1^2} = \sqrt{3}.$

So that in a right-angled triangle, in which one angle is 60°, the three sides of the triangle are proportional to 2, 1, and $\sqrt{3}$.

Also, area of triangle $ABC = \tfrac{1}{2} \times 2 \times \sqrt{3} = \sqrt{3}$ square units.

Ex. 3. Find the area of an equilateral triangle, each side 10 ft. in length.

$$\text{The area} = \tfrac{1}{2}(AB \times CD), \quad \text{(Fig 47)}$$
$$CD = \sqrt{10^2 - 5^2} = 8{\cdot}66$$
$$\text{Area} = \tfrac{1}{2}(10 \times 8{\cdot}66) = 43{\cdot}3 \text{ sq. ft.}$$

When the three sides of a triangle are given *

If a, b, c be the three sides and $s = \dfrac{a+b+c}{2}$; when s = half the sum of the sides, then the area of the triangle is given by the formula

$$\textbf{area} = \sqrt{s(s-a)(s-b)(s-c)},$$

or, find half the sum of the length of the sides, subtract from this half sum the length of each side separately; multiply the three remainders and the half sum together, the square root of the product is the area of the triangle required

Ex. 4. We may use this rule to find the area of a right angled triangle, sides 3, 4, and 5 units respectively The area has been already determined by another method in Ex 1

Here $\quad s = \tfrac{1}{2}(3+4+5) = 6$

Subtract from this the length of each side separately, *i.e.*

$$6-3 = 3, \quad 6-4 = 2, \quad 6-5 = 1$$

$$\text{Area of triangle} = \sqrt{6 \times 3 \times 2 \times 1} = \sqrt{36} = 6 \text{ sq. units.}$$

Ex. 5. Find the area of a triangle, the lengths of the three sides being 3·27, 4·36, and 5·45 feet respectively

$$s = \tfrac{1}{2}(3{\cdot}27 + 4{\cdot}36 + 5{\cdot}45) = 6{\cdot}54$$
$$\therefore \text{Area} = \sqrt{6{\cdot}54(6{\cdot}54 - 3{\cdot}27)(6{\cdot}54 - 4{\cdot}36)(6{\cdot}54 - 5{\cdot}45)}$$
$$= \sqrt{6{\cdot}54 \times 3{\cdot}27 \times 2{\cdot}18 \times 1{\cdot}09} = \sqrt{50{\cdot}8169}$$
$$= 7{\cdot}129 \text{ sq. ft}$$

Ex. 6. The sides of a triangular field are 500, 600, and 700 links respectively. Find its area.

$$\tfrac{1}{2}(500 + 600 + 700) = 900,$$
$$\text{area} = \sqrt{900 \times 400 \times 300 \times 200} = 146969 \text{ sq. links}$$
$$= 1 \text{ ac. } 1 \text{ r. } 35{\cdot}15 \text{ poles.}$$

* This useful result must at this stage be taken for granted. Its proof will come later.

EXERCISES. XXXIII.

1. The base of a triangle is 49 ft. and the height 25·25 ft., find its area.

2. Find the area in acres of a triangle whose sides are 25, 20, and 15 chains respectively.

3. Find the area of the triangle whose sides are 13 ft., 14 ft., and 15 ft.

4. Find the area of the triangle whose sides are 21, 20, and 29 inches respectively.

5. The sides of a triangular field are 350, 440, and 750 yds. The field is let for £31. 10s. per year; what is the rent per acre?

6. On a base of 10 yards a right angled triangle is formed with one side 2 yards longer than the other. Find its area

7. The sides of a triangle in yards are 101·5, 80·5, and 59·4 respectively, find the area.

8. Find the area of a triangle, base 625 links, height 1040 links.

9. Find the area in square yards of a right-angled triangle, the two perpendicular sides being 80 and 30 ft. respectively.

10. Find the area of a triangular field, the sides of which are 2569, 4900, and 5025 links respectively.

11. The sides of an equilateral triangle are each 17 chains 4 links Find the area.

12. A triangular field, the three sides of which are 350, 440, and 750 yards respectively, is sold for £52. 10s., find the selling price per acre.

13. At £5 per acre find the cost of a triangular field, the three sides being 130, 140, and 150 yards.

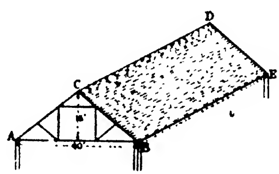

FIG. 48.

14. The span AB (Fig. 48) of a roof is 40 ft., the rise 15 ft.; find the total area covered by slating if the length of the roof is 60 ft.

15. The sides of a triangular field are 300, 400, and 500 yds. If a belt, 50 yds. wide, is cut off the field, what are the sides of the interior triangle, and what is the area of the belt?

16. Find the area of a triangle, the sides being 15, 36, and 39 ft. respectively.

17. The three sides of a triangle are 15, 16, and 17 ft. respectively, find its area.

18. Find the area of a triangular piece of land, in which the three sides are 600, 700, and 800 yds. respectively.

19. Find the area of an equilateral triangle, each side 10 ft. in length.

20. The sides of a triangular field are respectively 10 chains, 8 chains, and 12 chains. The chain being 22 yards, find the acreage of the field, and the perpendicular distance of the longest side from the opposite corner.

21. The area of a triangular field is 6 a. 2 r. 8 p., and the perpendicular from one angle on the base is 524 links. Find the length of the base.

22. The gable end of a house measures 9 ft. 7 in. in height from the base to the ridge. If the length of the base AB (Fig. 49) be 20 ft. 10·4 in., find the area.

Fig. 49.

23. The area of an equilateral triangle is 625·25 sq. ft., find the length of a side.

24. The area of a triangular field is 16 ac. 3 r. 8 p., the base 14 chains. Find the length of the perpendicular from the opposite angle to the base.

25. The sides of a triangle are 624, 780, and 468 links respectively. Find its area in acres.

Area of a Rhombus. Let $ABCD$ (Fig. 50) be a given rhombus. By joining A to C, the figure is divided into two triangles ACB and ACD. Also *the two diagonals of a rhombus bisect each other at right angles.* Moreover, the line IB is the altitude of the triangle ACB and ID is the altitude of the triangle ACD.

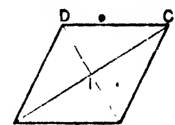

Fig. 50.—Area of a rhombus.

Therefore *area of rhombus*

$$= \text{area of triangles } ACD \text{ and } ACB$$
$$= \tfrac{1}{2}(AC \times IB) + \tfrac{1}{2}(AC \times ID)$$
$$= \tfrac{1}{2}(AC \times BD),$$

hence *the area of a rhombus is half the product of the two diagonals.*

When a diagonal AC and one side AD of a rhombus are given, then, since all the sides of a rhombus are equal in length, the area is obtained as follows:

In the triangle ADC the three sides AD, DC, and CA are known. From this, as on p. 131, the area of the triangle ADC can be obtained. The area of the rhombus is obviously twice the area of ADC.

Ex. 1. Find the area of a rhombus, the diagonals being 30 and 12 feet respectively.

$$\text{Area} = \tfrac{1}{2}(30 \times 12) = 180 \text{ square feet}$$

Ex. 2. If in a rhombus the length of one side AD is 65 and that of one diagonal is 104 units, find the area.

Here $\qquad \tfrac{1}{2}(65 + 65 + 104) = 117.$

$$\therefore \text{Area of triangle} = \sqrt{117 \times 13 \times 52 \times 52}$$
$$= 2028 \text{ units of area}$$

Hence area of rhombus $= 2028 \times 2 = 4056$ units of area.

Area of a Trapezium. The area may be obtained by dividing the figure into two triangles ACD and ACB (Fig. 51).

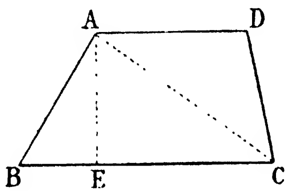

FIG. 51.—Area of a trapezium.

If AE be drawn perpendicular to BC, and meeting BC in E, then AE is the altitude of both the triangles ABC and ACD.

$$\text{area of triangle } ABC = \tfrac{1}{2}(BC \times AE),$$
$$\text{area of triangle } ACD = \tfrac{1}{2}(AD \times AE).$$

Area of four-sided figure $ABCD$ equals area of the two triangles into which the figure is divided by the line AC.

$$\text{area of } ABCD = \tfrac{1}{2}(BC \times AE) + \tfrac{1}{2}(AD \times AE)$$
$$= \tfrac{1}{2}AE(BC + AD).$$

This important result for the *area of a trapezium* may be stated as follows: ***Multiply the sum of the parallel sides by half the perpendicular distance between them.***

Ex. 1. **Find the area of a trapezium, the lengths of the two parallel sides being 650 and 925 ft. respectively, and the perpendicular distance between them 420 ft.**

$$\text{Area} = \tfrac{1}{2}(650 + 925)420$$
$$= 330750 \text{ sq. ft.}$$

Ex. 2. Find the perpendicular distance between the two parallel sides of a trapezium, the lengths being 74 and 84·8 ft respectively, and the area 762·24 sq. ft.

Here $\tfrac{1}{2}(74 + 84{\cdot}8) = 79{\cdot}4$,

$$\text{perpendicular distance} = \frac{762{\cdot}24}{79{\cdot}4} = 9{\cdot}6 \text{ ft}$$

Area of any Quadrilateral. Any four sided figure, such as $ABCD$ (Fig 52) is called a quadrilateral By joining D to B the figure is divided into two triangles ADB and DBC. The area of the quadrilateral is the sum of the areas of the two triangles. Draw AE and CF perpendicular to DB.

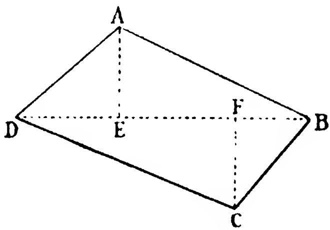

FIG. 52.—Area of a quadrilateral

Then area of triangle $ADB = \tfrac{1}{2}(DB \times EA)$

Also area of triangle $DCB = \tfrac{1}{2}(DB \times CF)$

$\therefore$ area of quadrilateral $= \tfrac{1}{2} DB(EA + CF)$,

or, *half the product of one of the diagonals and the sum of the two perpendiculars let fall on that diagonal from the two opposite angles gives the area of the quadrilateral.*

Ex. 1. The lengths of the sides of a quadrilateral figure are AB 107½ ft., BC 127½ ft., CD 150 ft., DA 62½ ft, and the length of the diagonal BD is 132½ ft. Find the area of the quadrilateral.

As the three sides of the triangles ABD and BCD are given, we

can find the area of each triangle separately and add the results to obtain the total area of the quadrilateral as follows:

Half the sum of the sides of the triangle DBC is

$$\tfrac{1}{2}(127{\cdot}5 + 150 + 132{\cdot}5) = 205,$$

subtracting from this 127·5, 150, and 132·5, respectively, we obtain 77·5, 55, and 72·5.

$$\therefore \text{ Area of triangle } DBC = \sqrt{205 \times 77{\cdot}5 \times 55 \times 72{\cdot}5}$$
$$= 7960 \text{ sq. ft.}$$

In a similar manner the area of the triangle DBA is

$$\sqrt{151{\cdot}25 \times 18{\cdot}75 \times 88{\cdot}75 \times 43{\cdot}75}$$
$$= 3318 \text{ sq. ft.}$$

$$\therefore \text{ Area of quadrilateral } DABC = 7960 + 3318$$
$$= 11278 \text{ sq. ft.}$$

Ex. 2. Let $DA = 48$ ft., $AB = 20$ ft., and $BD = 52$ ft., also let $CF = 40$ ft. Then $\tfrac{1}{2}(48 + 52 + 20) = 60$.

$$\text{Area of triangle } DAB = \sqrt{60 \times 12 \times 8 \times 40}$$
$$= \sqrt{230400} = 480 \text{ sq. ft.}$$
$$\text{Area of triangle } DCB = \tfrac{1}{2}(52 \times 40) = 1040 \text{ sq. ft.}$$
$$\therefore \text{ Area of quadrilateral} = 480 + 1040$$
$$= 1520 \text{ sq. ft.}$$

EXERCISES. XXXIV.

1. The parallel sides of a trapezium are 234 ft. and 104 ft. respectively, the perpendicular distance between them is 92 ft.: find the area of the trapezium.

2 The plan of a district is found to be a trapezium having its parallel boundaries 276 and 216 miles respectively, and its breadth 72·5 miles: find the area of the district.

3. A field in the form of a trapezium has its parallel sides 10 chains 30 links and 7 chains 70 links; the distance between them is 7 chains 50 links: find the area of the field.

4. $ABCD$ is a quadrilateral field in which the angles C and D are right angles, and the angle A is half a right angle. Find the area of the field, given that $BC = 91$ yds., and $AD = 151$ yds.

5. In a trapezium the parallel sides are 750 and 1225 links, and the perpendicular distance between them 770 links: find its area.

6. In a quadrilateral one diagonal is 84 ft., and the two perpendiculars on it from the other two angles are 16 ft. and 18 ft. respectively: find its area.

7. $ABCD$ is a four-sided field such that the lines joining opposite corners A, C, B and D meet at right angles at F; the lengths FA, FB, FC and FD measuring 83, 97, 125 and 238 yards respectively: find the area of the field.

8. A quadrilateral has two of its sides parallel; these sides are 10 and 12 ft. respectively; the perpendicular distance between them is 4 ft.: find the area of the figure.

9. The sides of a quadrilateral taken in order are 27, 36, 30, 25 ft; the angle between the first two is 90°: find its area.

10. The parallel sides of a trapezium are respectively 27·5 and 38·5 ft.: the perpendicular distance between them is 12·385 ft.: find its area.

11. Two sides of a field, which are parallel, are 7 chains 33 links and 16 chains 19 links respectively in length, and the perpendicular distance between them is 27 chains 8 links: find the area of the field in acres.

12. An earthwork in the form of a trapezium has a breadth of 12 ft. at the top and 38 ft at the bottom, its height being 12 ft.: find the number of cubic feet in a rod of its length

13. The two parallel sides of a field are 650 and 925 links respectively, and the perpendicular distance between them is 420 links: find the area of the field.

14. The two parallel sides of a field are 700 links and 300 links respectively; if the perpendicular distance between them be 620 links, find the area of the field.

15. The distance between the two parallel sides of a trapezium is 19 yds., and one parallel side is 4 yds. longer than the other: find the length of each side when the area is 475 sq yds.

16. $ABCDE$ is a five sided field. The side AB is 533 links, the diagonal BD is 875 links, and diagonal AD is 888 links. The perpendicular from C on to BD is 360 links, and the perpendicular from E on to AD is 700 links. Find the area of the field in acres.

Summary.

Area of a Triangle = ½ base × altitude Or, from half the sum of the three sides subtract each side separately, multiply the half sum and three remainders together; the square root of the product gives the area of the triangle.

Area of a Rhombus = *half the product of the two diagonals.*

Area of a Trapezium = Multiply *half the sum of the parallel sides by the perpendicular distance between them.*

Area of a Quadrilateral. Join any two opposite corners by a diagonal line; on this diagonal let fall perpendiculars from the remaining two opposite angles and obtain the lengths of these perpendiculars Then the area of the quadrilateral is *half the product of the diagonal and the sum of the two perpendiculars.*

CHAPTER XV.

MENSURATION OF HEXAGON AND OF ANY POLYGON.

Area of a Regular Hexagon.—In the case of a regular six-sided figure, that is, a hexagon, as in $ABCDEF$ (Fig. 53), the area can be obtained by dividing the figure into as many triangles as there are sides to the figure. All these triangles are equal in area, and the area of the hexagon is 6 times that of the triangle OAB.

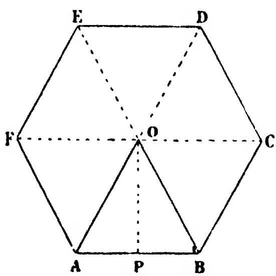

FIG. 53.—Area of a hexagon.

Let a denote the length of the side AB. With any centre O and radius $OA = a$, describe a circle. The length OA, it will be found, may be marked off exactly 6 times along the circumference, so obtaining points A, B, C, D, E, F. Join A to B, etc., as in Fig. 53; then $ABCDEF$ is a regular hexagon.

Join the centre O to A and B. Then OAB is an equilateral triangle.

If OP be a line perpendicular to AB, and passing through O;

$$AP = PB = \frac{a}{2}.$$

Also $$OP = \sqrt{a^2 - \frac{a^2}{4}} = \frac{a}{2}\sqrt{3}.$$

Hence, area of triangle $OAB = AP \times OP = \frac{a}{2} \times \frac{a}{2}\sqrt{3}$.

$$\therefore \text{ area of hexagon } = 6 \times \frac{a^2}{4}\sqrt{3} = \frac{3}{2}a^2\sqrt{3}.$$

The value of $\sqrt{3}$ is known to be 1·732 (p. 46),

$$\therefore \text{ area of hexagon } = 2·598 \times a^2,$$

or, to find the *area of a hexagon, square the side and multiply by* 2·598.

Using this rule when the area is given, the length of side can be obtained.

Ex. 1. The length of each side of a regular hexagon is 4 feet, find its area.

$$\text{Area} = 2·598 \times 4^2 = 2·598 \times 16$$
$$= 41·568 \text{ sq. ft.}$$

Ex. 2. The area of a regular hexagon is 1039·2 sq. in., what is the length of the side?

We have $1039·2 = a^2 \times 2·598$,

where a denotes the side of the hexagon.

$$\therefore a^2 = \frac{1039·2}{2·598} = 400$$
$$\therefore a = 20$$

$\therefore$ Length of side = 20 in.

EXERCISES. XXXV.

1. The area of a regular hexagon is 7 sq. ft. 31·228 sq. in., find the length of a side.
2. The side of a regular hexagon is 10·74 yards, find its area.
3. Find the side of a regular hexagon, equal in area to an equilateral triangle, whose side is 150 ft.
4. The length of each side of a hexagon is 12 ft., find its area.
5. Find the area of a hexagon whose side is 20 ft.
6. The area of a hexagon is 286·437 sq. ft., find the length of a side.

7. The area of a hexagon is 10·39 sq. chains, find the length of one of its sides.

8. The area of a hexagon is 3367·699 sq. yds., find the length of its side.

Area of any Polygon.—The area of any irregular polygon may be found by dividing the figure into a number of triangles, then by finding the area of each triangle, and finally adding together all the areas so obtained, the result is the area of the figure.

Graphically. The area of an irregular figure may be obtained graphically as follows

Let $ABCDE$ (Fig. 54) be an irregular five-sided figure.

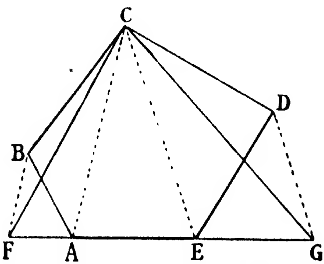

Fig. 54.—Graphical method of finding the area of a polygon.

Join C to A, and from B draw a line BF parallel to CA, meeting EA produced at F. Join C to F.

The triangles CFA and CBA are on the same base CA, and have the same altitude. Hence the triangle CFA is equal in area to the triangle CBA.

In a similar manner, join C to E, and from point D draw DG parallel to CE, meeting AE produced at G. Join C to G.

The triangles CGE and CDE are equal in area.

Thus the triangle FCG is equal in area to the given five-sided figure. As the length of the base FG, and also the altitude of the triangle can be obtained, the area of the triangle, and therefore the area of the given five-sided figure can be ascertained.

Or, the triangle FCG may be converted into an equivalent rectangle, as on p. 130.

MISCELLANEOUS EXERCISES XXXVI.

1. If the base of a rectangle is 1 mile in length, and the diagonal $1\frac{1}{4}$ miles, find the area.

2. Find the cost of putting a fence, at 1s. 6d. per yard, round a square field whose area is 4 acres.

3. Find the difference in the cost of papering the walls of a room 19 ft. 4 in. long, 12 ft. 2 in. wide, and 20 ft. 6 in. high, with paper 21 in. wide and costing 7d. a yard, or with paper 18 in. wide costing $6\frac{1}{4}$d. a yard, allowing 13 square yards for windows, etc.

4. The width of a lawn tennis ground is two thirds of its length. The cost of levelling it at 8d. per square yard is £115 4s.; find the cost of enclosing it with an iron railing at 6s. 8d. per yard.

5. Find the area of a path 3 ft. wide round a house 57 ft. long and 37 ft. wide.

6. A roof which measures 18 feet along the ridge, and 16 ft. 8 in. from ridge to eaves, is to be covered with slates 16 in. long and 8 in. wide, each slate overlaps the one below by 2 in.; find the number of slates required, and the cost at 2s. per dozen.

7. The diagonal of a rectangular field is to its length as 13 to 12, and its area is 1 ac. 20 sq. yds.; what is its breadth?

8. The perimeter of a rectangular field is to its length as 34 to 13, and the length exceeds the breadth by 70 yds.; what is its area?

9. A rectangular tank 3 ft. in width by 4 ft. in length receives all the rain that falls on a roof, length 30 ft., width 20 ft. On a certain day it was estimated that a pint of water fell on each square foot of roof; how far will the water rise in the tank?

10. Find the area of a rectangle, length $9\frac{2}{3}$ ft., width $5\frac{1}{3}$ ft.

11. Find the length of carpet $\frac{3}{4}$ yard wide necessary to cover the floor of a rectangular room $22\frac{1}{2}$ ft. long, $21\frac{1}{4}$ ft. wide.

12. Find number of rolls of paper required to paper the walls of a room 18 ft. long, 12 ft. wide, and $10\frac{1}{2}$ ft. high; each roll is 21 in. wide, and 12 yds. long.

13. The area of a rectangular plot is 101 sq. yds. 0 sq. ft. 8 sq. in.: its length is 36 ft. 9 in.; find its breadth.

14. The length and width of a rectangular tile are 12 in. and 9 in. respectively; find the number required to pave a passage 90 ft. long by 9 ft. wide.

15. How many cwt. of lead will be required to cover a roof 48 ft. long and 32 ft. wide with "5 lb. lead"?

16. Find the cost of the paper for a room 17 ft. 7 in. long by 11 ft. 5 in. wide, and 10 ft. high, using paper 2 ft. 8 in. wide at 8d. per yard.

17. The sum of £9 0s. 10d. is allowed for the paper for a room 27 ft. long, 19·55 ft. wide, and 12·4 ft. high; how much per yard must be given for a paper 2·7 ft. wide?

18. The length of a hall is three times the breadth: the cost of whitewashing the ceiling at 5¼d per sq. yd. is £4 12s. 7·1d., and the cost of papering the four walls at 1s. 9d. per. sq. yd. is £35; find the height of the hall.

19. A rectangular field, four times as long as it is broad, contains 2½ acres; find its length and breadth.

20. Find the cost of painting the four sides and the bottom of a tank 3½ yds. long, 3½ ft. wide, and 6 ft. deep, at 4d. per square foot

21. A room is 18 ft long, 12 ft. wide, and 11 ft. high; what length of paper a yard wide would be required to line its four walls and its ceiling?

22. There is a square enclosure of 10 acres; a man walks at the rate of 3 miles an hour along one side, along a diagonal, along another side, and so returns along the other diagonal to the starting point: how many minutes does it take him to walk the distance?

23. The length of a floor is 28 ft. and width 19 ft.; find the cost of carpeting it with carpet ¾ yard wide, and costing 5s. 9d. a yard.

24. The length of a field is ¼ mile, the width 242 yards; find its area.

25. The length of a rectangle is 81 ft., its width $5\frac{4}{9}$ ft.; find the side of a square, the area of which is equal to that of the rectangle.

26. Find the cost of fencing a square field of 10 acres at 9d. per yard.

27. The length of a courtyard is 20 yds. 2 ft 6 in., and width 12 yds. 2 ft. 3 in.; find the number of stones required to pave it, each stone being 2 ft. in length and 1 ft 5 in. wide.

28. Find the area of a triangular field, the three sides being 848, 900, and 988 links respectively.

29. The two parallel sides of a trapezium are 750 and 1225 links respectively, the perpendicular distance between them 1540 links; find its area.

30. A lawn-tennis ground is half as long again as it is wide: the cost of levelling it at 9d. per square yard is £176. 8s.; find the cost of enclosing it with an iron railing at 7s. 6d. per yard.

Summary.

Area of a Regular Hexagon.—The area is the product of 2·598 and the square of one of the sides.

Area of any Polygon.—The figure may be divided into a number of triangles; the area of the polygon is then the sum of the areas of all the triangles.

MISCELLANEOUS EXERCISES.

Section I. Vulgar Fractions.

1. Explain why $\frac{2}{3}$ of $\frac{3}{4}$ is equal to $\frac{1}{2}$.

2. Reduce the following fraction to its simplest form:

$$\frac{(9\frac{1}{[illegible]} \text{ of } 4\frac{[illegible]}{7}) \cdot 8\frac{1}{4}}{2\frac{[illegible]}{[illegible]} \cdot \frac{[illegible]}{12} + 1\frac{[illegible]}{16}}$$

3. Of a builder's stock of timber $\frac{2}{[illegible]}$ was destroyed by fire and $\frac{4}{15}$ damaged. What fraction of his stock remains uninjured?

4. $\frac{1}{3}$ of a pole is under water and $\frac{1}{5}$ is in the mud at the bottom. What fraction of the pole is above the surface of the water?

5. An apprentice spends $\frac{[illegible]}{[illegible]}$ths of his time in the shop and $\frac{3}{8}$ths in the drawing office. What fraction represents the remainder of his time?

6. Find the value of $\frac{3}{4}$ of the sum of £1692 19s. 9d. and £19. 6s. 11d.

7. Express the ratio of 17 hours 45 minutes 28 seconds to 124 hours 18 minutes 16 seconds as a vulgar fraction in its lowest terms.

8. The cost of labour in producing a certain article was £18. 19s. 11d. It was made by five persons, who severally spent two, three, four and a half, six and eight days upon it. How should the money be divided among them?

9. What part of a log remains after $\frac{1}{12}$, $\frac{3}{8}$ and $\frac{1}{4}$ of it have been cut off?

10. A farm consists of 300 acres, $\frac{2}{15}$ of it is pasture and the rest is arable. How many acres are arable?

11. Coals at 32s. per ton are mixed with coke at 24s. a ton in the proportion of 5 tons of coal to 3 tons of coke; find the money saved by using 11 tons of the mixture instead of 12 tons of coal.

12. The pressure of the atmosphere is 14 7 lbs. per sq. in.; what is the pressure in lbs. per square foot and in kilograms per sq. metre?

13. (i) Subtract $\frac{6}{10}$ of $\frac{5}{7}$ of $\frac{7}{3}$ from $\frac{5}{13}$ of $5\frac{4}{7}$.

(ii) Divide $\dfrac{2\frac{1}{4}}{9\frac{3}{7}}$ by $\frac{1}{2}$ of $\frac{6}{11}$.

14. A man buys 120 articles for 5s. 6d.; he sells one-third of them at a penny each, one-fifth of them at two for $1\frac{1}{2}$d. and the rest at $\frac{1}{2}$d. each. Find how much profit he makes.

15. Divide £793 10s. amongst *A*, *B*, *C* and *D*, so that *A* may have two-thirds as much as *B*, whilst *B* has three-fourths as much as *C*, and *C* has five-sixths as much as *D*.

16. If $\frac{8}{19}$ths of *A*'s capital is $\frac{2}{3}$rds of *B*'s capital, and if the capitals of *B* and *C* are taken together, they amount to $\frac{10}{11}$ths of *A*'s capital, find what part of *B*'s capital equals *C*'s capital. Find three whole numbers which are proportional to the three capitals.

17. Find which is greater, $\frac{2}{7} + \frac{5}{9}$ or $\frac{12}{11} - \frac{3}{13}$.

18. The rates amount to £11. 3s. $11\frac{1}{2}$d. for a house whose rateable value is £62. 10s.; find what they will amount to, on the same scale, for a house whose rateable value is £105.

19. One kind of gunpowder is made of 15 parts nitre, 2 parts sulphur and 3 parts charcoal; another kind is made of 77 parts nitre, 9 parts sulphur and 14 parts charcoal. If 112 lbs. of each kind be mixed together, find the amount of each of the three constituents in the mixture, giving the answer in pounds.

Section II. Decimals.

Add together

1. 26·9872, 324·09885, 0·123 and 4·26.

2. 4·79093, 59·6358, 0·002503 and 5·7015151.

3. 0·79093, 59·6358, 0·002503 and 346·0207.

4. 379·8503, 0·0056178, 5·973 and 1·50034621.

5. 181·5276, 10·0085, 0·16709 and 11·962.

Subtract

6. 72·807236 from 631·04357.

7. 29·78787 from 31·010101.

8. 549·8625 from 782·296.

9. 9·099901 from 10·00901.

10. 12·0360819 from 107·00817.

11. 23·907325 from 84·0157.

12. Add together 346·07643, 3·974928 and 0·00647328, and subtract the result from 359·038622.

Compute by contracted methods to four significant figures:

13. $0·03405 \times 0·9123$.

14. $7·164 \times 0·00734$.

15. $87·29 \times 0·01785$.

16. $12·39 \times 5·024$.

17. $8·102 \times 35·14$.

18. $3·405 \times 9·123$.

19. $23·07 \times 0·1354$.

20. $0·01239 \times 0·5024$.

21. $0·03405 \times 0·9123$.

22. $34·05 \div 0·09123$.

23. $0·01584 \div 2·104$.

24. $0·00729 \div 0·2735$.

25. 5·024 ÷ 12·39.

26. 254·3 ÷ 0·00027.

27. 3·405 ÷ 9·123.

28. 2307 ÷ 1·354.

29. 0·1239 ÷ 50·24.

30. 34·05 ÷ 0·09123.

Compute

31. (i) 1·353 × 0·2308. (ii) 23·08 ÷ 0·01353.

32. (i) 23·08 × 0·1353. (ii) 2308 ÷ 1·353.

33. (i) 12·39 × 5·024. (ii) 5·024 ÷ 12·39.

34. Add $\frac{2}{3}(\frac{3}{11} - 2)$ to $\frac{3}{4} \times 0\cdot0603 \div \frac{5}{4} \times 0\cdot00594$.

35. To $(2\cdot5 + 3\cdot625 - \frac{4}{5} + \frac{1}{7\frac{1}{2}})$ add $4\frac{1}{5} \times \frac{10}{63}$, and from the sum subtract $2\frac{3}{14} \div 8\frac{6}{7}$.

36. To $(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8} - 1\cdot25 + 3\cdot5)$ add $\frac{1}{52}$ of $4\frac{1}{3}$, and from the sum subtract $5\frac{1}{8} \div 1\frac{7}{8}$.

37. Subtract 0·0625 of £113. 16s. 8d. from $\frac{23}{24}$ of £50. 5s. 6d., and find the number by which the result must be multiplied to produce £985. 11s. 6d.

38. Express 6 cwt. 3 qrs. $3\frac{1}{2}$ lbs. as a decimal of a ton.

39. Find what decimal part of £78. 8s. 8d. is equal to £63. 14s. $0\frac{1}{2}$d.

40. A piece of wire 4·7 inches long weighs 55 grains; find the weight of a mile length of this wire. [1 lb. = 7000 grains.]

41. Express in its simplest form

$$\frac{1\frac{5}{7} \text{ of } £2.\ 11s.\ 11d.}{1\cdot5 \text{ of } £4.\ 18s.\ 8d. + 0\cdot3 \text{ of } £5}$$

42. Simplify

$$\frac{\frac{1}{2} - \frac{1}{3} + \frac{1}{8}}{\frac{1}{2} \text{ of } \frac{1}{3} \text{ of } \frac{1}{8}} \times 0\cdot21.$$

43. What is meant by 2·307 of a whole number? Find the value of 2·307 of 16 cwt. 3 qrs. 13 lbs.

44. A blotter cost 0·03 of a crown, a book 0·03 of a pound, and a penholder 0·03 of a shilling. What will be the total cost of the three articles?

45. Find what fraction £16. 1s. $10\frac{1}{2}$d. is of £59. 0s. $2\frac{1}{2}$d.; express the fraction in its lowest terms as a vulgar fraction, and then convert it into a decimal.

Simplify:

46. (7·3 × 0·0143 ÷ 15·015) − (0·152 × 0·033 ÷ 2·09).

47. (21·7 × 0·087 ÷ 2·03) + (102·01 × 0·319 ÷ 2·639).

48. Reduce to a decimal true to three places:

$$\frac{\frac{3}{11} \times 25\cdot15}{4\frac{6}{11} \times 0\cdot4}.$$

Section III. Involution and Evolution.

1. Find the square of 3·057. Also find the square root of 3·057 in each case to four significant figures.

2. Find to four significant figures the square roots of:

(i) $8\frac{1}{3}$, (ii) 0·144, (iii) 0·51, (iv) 0·051, (v) 0·571.

3. Find to four significant figures the square root of 17·375.

4. Divide the cube of 0·29 by the square of 0·058.

5. Multiply the cube of 3·25 by the square root of $7\frac{1}{9}$.

6. The length of the side of a square is 7·24 ft.; find the length of a diagonal.

7. Extract the square root of 4·001, and show that it is very little less than $2 + \frac{1}{4000}$.

8. Show that the square root of 7 is intermediate between 2·645 and 2·646, and find to the fifth place of decimals by how much the square root of 7 differs from 2·6455.

9. The length of one side of a square garden plot is 36 ft. Find the area of the plot in square yards.

10. The sides of a rectangle are 972 yds. and 1296 ft. Find the side of a square, the area of which is twice that of the rectangle.

11. A piece of sheet metal in the form of a right-angled triangle has its two perpendicular sides 4 ft. and 3 ft. respectively. Find the length of the third side.

12. The foot of a ladder 30 ft. long is 5 ft. from a vertical wall. How far up the wall will the ladder reach?

13. The cost of the gravel for a square courtyard at $2\frac{1}{2}$d. per sq. yd. is £33. 17s. Find the length of a side of the courtyard.

14. Two railroads cross each other at right angles at a point *P*. Two trains *A* and *B* are moving on them with velocities of 30 and $22\frac{1}{2}$ miles per hour respectively; at a given instant the front of *A* is 32 yards short of *P*, and the front of *B* is 100 yards past *P*; find how far the two fronts are apart at the end of three-fifths of a minute from the given instant. [$\sqrt{2} = 1·414$.]

Section IV. Averages and Ratio.

1. A train makes a journey of 89·6 kilometres in 1 hour 10 minutes. Find its average speed in centimetres per second.

2. What is meant by ratio? How would you express the ratio of 5 tons to 8 lbs.?

3. Express in their simplest forms

(*a*) The ratio of $2\frac{1}{4}$ to $7\frac{1}{3}$.

(*b*) The ratio of $\frac{3}{4}$ of 53 cwts. 3 qrs. 3 lbs. to 0·4 of 65 cwts. 0 qrs. 11 lbs.

4. Find the average of 29·05, 28·95, 28·45, 28·04, 28·62, 29·00, 29·85.

5. Divide 204 into three parts proportional to the numbers 7, 8, 9.

6. Find the ratio of £2 17s. 6d. to £12 18s. 9d., and the ratio of 2 cwts. 2 qrs. 10 lbs. to 12 cwts. 0 qrs. 19 lbs. Find the difference between the ratios.

7. How many pounds are there in 0·708624 of a ton? How many kilograms are there in it if 100 kilograms equal 1·9684 cwts.

Section V. Percentage.

1. 10 cwt. of sugar is sold at $2\frac{1}{4}$d. a pound, thereby gaining 11s. 8d. What is the profit per cent.?

2. A contractor finds that out of 2000 bricks $\frac{1}{4}$ are useless and 65 are broken. What percentage of the whole number are sound?

3. A motor car was bought for £850, the average working expenses are £16 per month, the cost of repairs, etc., per annum is £75. What percentage of the cost of the car is the total annual expense?

4. If the air in a room contains 20·1 per cent. by volume of oxygen, find the number of cubic feet of this gas present in a room which measures $16\frac{1}{2}$ feet every way.

5. Suppose a contractor loses 33 per cent. of his money and then finds he has £2000 left. How much had he at first and what did he lose?

6. A grocer has two kinds of tea which cost him 2s. 8d. and 2s. 1d. per lb. respectively. Find how many lbs. of the former kind he must mix with 56 lbs. of the latter so that he may gain 25 per cent. by selling the mixture at 2s. 11d. per lb.

What is meant by percentage?

7. Divide £814 among three persons in the ratios $\frac{2}{5}$, $\frac{3}{7}$ and $\frac{5}{8}$. State what percentage of the total amount each person receives.

8. One kind of gunpowder consists of 9 per cent. sulphur, 14 per cent. charcoal and 77 per cent. nitre. Another kind is made of 2 parts sulphur, 3 parts charcoal and 15 parts nitre. If 200 lbs. of each be mixed together, find the amount of each of the three constituents in the mixture.

9. A sum of £2928 is to be divided among four persons in the ratios $\frac{2}{3} : \frac{3}{4} : \frac{4}{5} : \frac{5}{6}$. Find the share of each.

State the percentage of the total amount which each person receives.

10. It is found that a motor car covers a measured distance of 100 yards in 9·74 seconds. What is the speed in miles per hour? If the estimation of the time is $\frac{3}{4}$ of a second in excess, what would be the speed? Find the percentage error.

11. A shopkeeper marks his goods at a price from which he can deduct $7\frac{1}{2}$ per cent. for prompt payment, and still have a profit of 10 per cent. on what the goods cost him. Find the cost price of an article which he marks at £2 15s.

12. A merchant buys 90 tons of coal at 17s. 6d. per ton; he sells 40 tons at 21s. 6d per ton and 50 tons at 23s. 6d. per ton. Find the profit per cent.

13. A merchant buys a quantity of iron ore and sells a part of it at a profit of 25 per cent.; he sells the remainder at a profit of 12 per cent; on the whole he thereby makes a profit of 18 per cent. What part of the original quantity did he sell at the higher price?

14. A sum of £1000 was invested in the purchase of $2\frac{1}{2}$ per cent. stock at $88\frac{3}{4}$. Find how much stock was obtained and the annual income that will be received.

What would have been the income if the £1000 had been invested in a $3\frac{1}{2}$ per cent. stock at 104?

15. If copper ore costs £3. 19s. 3d. per ton and yields $14\frac{7}{8}$ per cent. of pure copper, find the cost of the ore required to produce one ton of copper.

16. Divide £56 between *A*, *B*, *C* and *D* in the ratios of the numbers 3, 5, 7, 9. State what percentage of the total amount each person receives.

17. A quantity of ore containing 23 per cent. of copper is bought at 9s. per cwt., and 95 per cent. of the copper is extracted at a cost of 2s. $10\frac{1}{2}$d. per cwt. of ore. Find the price per ton at which the copper must be sold if a profit of 15 per cent. is to be made.

Section VI.

1. If the price of meat in France is 2·75 francs per kilogram, what is the price per pound? [5 kilos. = 11 lbs. 1 franc = $9\frac{3}{5}$d.]

2. Find the value in Napoleons and francs of £25. 16s., the exchange being 25·6 francs per £1. [1 Napoleon = 20 francs.]

3. Express a pressure of 1 ton per square foot in kilograms per square metre. [1 kilogram = $2\frac{1}{5}$ lbs. 1 metre = 39·37 ins.]

4. Supposing a farmer spreads two tons of lime on an acre of ground, how many ounces would this be to the square yard, and how many grams to the square metre?

5. If in Paris a sovereign can be exchanged for 25 francs 10 centimes, and in London a 20-franc piece can be exchanged for 15s. 4d., find what is gained or lost by changing £25 in Paris into French money, and then changing the amount received back into English money in London.

6. How many pounds and how many kilograms are there in 0·308624 of a ton? How many kilograms are there in it if 100 kilos. equal 220·46 lbs.?

Section VII. Algebra.

1. Let x be multiplied by the square of y, and subtracted from the cube of z; the cube root of the whole is taken and is then squared. This is divided by the sum of x, y and z. Write this algebraically.

2. Write down algebraically: Add twice the square root of the cube of x to the product of y squared, and the cube root of z. Divide by the sum of x and the square root of y. Add four and extract the square root of the whole.

3. Write down algebraically: Square a, divide by the square of b, add 1, extract the square root, multiply by c, divide by the square of n.

4. Subtract b^3 from a^3 and divide the result by the sum of a and b. (i) Express this algebraically in its simplest form. (ii) Find its numerical value when $a = 8·352$, $b = 2·742$.

5. When x and y are small, then $\frac{1+x}{1+y}$ is very nearly equal to $1 + x - y$.

What is the error in this when $x = 0·02$ and $y = 0·03$?

6. State what are the values of $a^3 \cdot a^5$ and of $(a^3)^4$, and justify your answers when $a = 2$.

7. Subtract $3a - 5b + 2c$ from $-4a - 2b + c$. Verify your answer by supposing that $a = \frac{1}{2}$, $b = \frac{2}{3}$, $c = \frac{1}{4}$, and finding the value of each of the quantities given and their difference.

8. State what is meant by a^7 and explain why $a^4 \times a^3 = a^7$.

9. Find the value of

$$-[a - \{b - (c - x) - (b + x)\} + d]$$

when a, b, c, d are equal to each other.

10. Reduce to its simplest form

$$(x + 2y + z)(x + 2y - z) - (x - 2y + z)(x - 2y - z).$$

11. Find the values of the following expression:
When $a=7$, $b=8$, $c=10$,

$$\text{(i)}\quad \frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)};$$

$$\text{(ii)}\quad \frac{a-b}{(b-c)(c-a)}+\frac{b-c}{(c-a)(a-b)}+\frac{c-a}{(a-b)(b-c)}.$$

12. Find the sum of

$$a+2b-3c,\quad -3a+b+2c \text{ and } 2a-3b+2c.$$

Verify your answer by supposing that $a=\frac{1}{3}$, $b=\frac{1}{2}$, $c=\frac{1}{4}$, and finding the value of each of the quantities given above and their sum.

13. Show that

$$4a^2b^2+2(a^2+b^2)(a+b)^2=(a+b)^4+(a^2+b^2)^2.$$

Verify the identity in the case when $a=2$, $b=3$.

14. Given $3x=1$, $2y=3$, $z=4$, find the numerical values of the following expressions:

$$\text{(i)}\quad 6xy-4yz+2zx;$$

$$\text{(ii)}\quad \frac{9x}{8y^3}+\frac{24y}{z^2}+\frac{z}{36x^2};$$

$$\text{(iii)}\quad \frac{3x}{y+z}+\frac{z}{x+y}.$$

15. Given $2x=3$, $4y=3$ and $z=-2$, find the numerical value of

$$\text{(i)}\quad \sqrt{(8y+2z+7)}+\sqrt{(6x-8y+z)};$$

$$\text{(ii)}\quad \frac{x}{y}+\frac{4x+2z}{2x-z};$$

$$\text{(iii)}\quad xy+yz-zx.$$

16. If $2a=3$ and $4b=5$, find the numerical values of:

$$\text{(i)}\quad \frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}}+\frac{\frac{2}{b}}{\frac{3}{a}+\frac{4}{b}};\qquad \text{(ii)}\quad \sqrt[3]{\frac{30a^2b^2}{a+b}};\qquad \text{(iii)}\quad \sqrt{\frac{3a^2-ab-2b^2}{(2b-a)(3a+8b)}}.$$

17. Find by how much the quantity

$$\left(1-\frac{2px}{x^2+px+q}\right)\times\left(1+\frac{3px}{x^2-px+q}\right)$$

exceeds unity, and verify your answer for the case in which $p=2$, $q=3$, $x=1$.

18. Given
$$x=\frac{w+\sqrt{(2W^2+w^2)}}{2W}$$
and
$$R=\sqrt{\{W^2-w\sqrt{(2W^2+w^2)}\}}.$$
If $W=10$, $w=5$, find x and R.

19. $$P = \frac{\mu W}{\cos\theta + \mu \sin\theta}.$$

Given $\mu = 0.3$, $W = 10$, $\cos\theta = 0.866$, $\sin\theta = 0.5$, find P.

20. $$x = \sqrt{\frac{2gU}{W}}, \quad y = 2\sqrt{\frac{Wr}{m+n}},$$

where $g = 32.2$, $U = 100$, $W = 30$, $r = 3$, $n = 2$, $m = \frac{W}{g}$. Find the numerical values of x and y and $x + y : x$.

21. Given $$R = \frac{\sqrt{3} - 1}{2\sqrt{3}}W.$$

Find the numerical value of R if $W = 100$.

22. $$S = Vt + \tfrac{1}{2}ft^2.$$

Find S when $V = 50$, $t = 2$, $f = 16$.

23. $$P = \frac{W(1+m)^3}{3 + 3m + m^2}.$$

If $W = 1000$, $m = 0.2$, find P.

Section VIII. Multiplication and Division.

1. Multiply $x^4 - 2x^3y + 3x^2y^2 - y^4$ by $x^2 - 2xy - 3y^2$.

2. Divide $x^3 - 6x^2 + 5x - 12$ by $x - 3$, and use your work to factorize the dividend.

3. Find the square of $x^4 - x^2 + 2x - 3$.

4. Multiply $x^5 + x^4 + x^3 + x^2 + x + 1$ by $x - 1$.

5. Multiply $x^3 - 3x^2y - 3xy^2$ by $x^2 - 5xy + 2y^2$. Verify the result by supposing that $x = 2$ and $y = 1$, and substituting these values in each of the two given expressions and that which you have found for their product.

6. Multiply $2x^3 - 3x^2y + 4xy^2 - 5y^3$ by $2x^2 + 3xy + 4y^2$, and find the value of the product when $x = 1$, $y = 0.1$.

7. Multiply $5a^2b(m+n)$ by $-6ac(m+n)^2$, and divide the product by $-15abc(m+n)^3$.

Section IX. Simple Equations.

Solve the following equations.

1. $3x - 2 = 2x + 2$.

2. $3x + 3 = 9x - 9$.

3. $6x + 2 = 6 - 2x$.

4. $\frac{2x-5}{4} - \frac{3x-11}{5} + \frac{x-3}{6} = 0$.

5. $(x - \frac{5}{2})(x + \frac{3}{2}) - (x-5)(x+3) = 12$.

Solve the following equations:

6. $(x+7)(x-8)=(x-3)(x+4)-48$.

7. $\dfrac{x-3}{3x+1}+\dfrac{2x-1}{4x-3}=1$.

8. $\dfrac{x-a}{b}-\dfrac{a^2+b^2}{ab}=\dfrac{x-b}{a}$.

9. $\left(x+\dfrac{1}{2}\right)\left(x-\dfrac{1}{3}\right)=x^2-\dfrac{x}{5}$.

10. $\dfrac{1}{x-a}+\dfrac{1}{x-b}=\dfrac{2}{x}$.

11. $\dfrac{x+a}{2}+\dfrac{x-b}{3}=a+b$.

12. $x-\dfrac{ax}{a+b}=\dfrac{ab}{a-b}-\dfrac{b^2x}{a^2-b^2}$.

13. $\frac{1}{3}(2x-10)-15=\frac{1}{11}(3x-40)-\frac{1}{5}(57-x)$.

14. $\dfrac{1}{x}+\dfrac{1}{2x}-\dfrac{1}{3x}=\dfrac{7}{3}$.

15. $(x-a)(x-b)-(x-c)(x-d)+\frac{1}{2}(a^2+b^2-c^2-d^2)=0$.

Section X. Simple Equations.

1. If one part of £400 be put out at 4 per cent. per annum, and the other part at 5 per cent. per annum, and if the yearly income obtained be £18. 5s., find the parts.

2. A sum of £23. 14s. is to be divided between A, B and C; if B gets 20 per cent. more than A and 25 per cent. more than C, how much does each get?

3. The sides of a triangle ABC are together 61 miles long; BC is $\frac{5}{8}$ of AB and three miles longer than CA; find the lengths of the sides severally.

4. A man pays away half his money to A, a third of what he has left to B and a fifth part of what he still has left to C; if after these payments he has 12s. 8d. left, how much had he at first?

5. A sum of money is divided between A, B and C; C gets twice as much as A: A and B together get £50; B and C together get £60; find the sum of money, and how much each person gets.

6. A certain number of persons ventured equal shares in a business, so as to make up a total capital of £1719. 8s. 4d. The first dividend was $7\frac{1}{2}$ per cent. upon the capital, and amounted to £2. 14s. $10\frac{1}{2}$d. per share. What was the number of shares?

7. Four lbs. of tea and 8 lbs. coffee cost together 12s. 6d.; if the tea became 20 per cent. dearer and the coffee 15 per cent. cheaper, the cost would be 13s. 3d. Find the price of a pound of each.

8. The length of a line A is nine-tenths of that of another line B, and a sixth part of A is 2·01 ft. longer than the tenth part of B; find the lengths of A and B.

9. A cyclist rode a journey at a constant speed of $14\frac{3}{4}$ miles an hour. If he had ridden 3 miles an hour slower, he would have been 64 minutes longer on the journey; how far did he ride?

10. *A* and *B* have between them £400; *A* receives a legacy of £350 and then he has twice as much as *B*; find how much *A* and *B* had at first.

Section XI. Mensuration.

1. Find the area of a parallelogram in which one of the diagonals is 480 ft., and each of the perpendiculars on it from the opposite angle is 100 ft.

2. The areas of a rectangular and a square field are equal, the lengths of the sides of the former being 135·5 and 103·8 metres respectively. Find the cost of enclosing the square field at 2·5 francs per metre.

3. The length of a rectangular field is to its breadth as 3 to 2, and its area is 1104 square metres. Find the sides and the cost of surrounding it with a fence at 2·25 francs per metre.

4. A map is drawn to a scale of 50 inches to a mile; how many square inches on the map will represent 10 acres on the ground?

5. A rectangular plot of ground measures 300 acres, its length being three times its breadth.

Find to the nearest yard the length of fence which would be required to enclose it.

6. The sides of a rectangle are 16 ft. and 10 ft. long respectively. Find the length of the diagonal of a square, the area of which equals that of the rectangle.

7. The area of a country is 32,300,000 acres; it consists of three kinds of land, viz., first, arable and garden; secondly, meadow, pasture and marsh; thirdly, waste; the areas of these kinds are in proportion to the numbers 2, 3 and $\frac{2}{3}$. How many acres are there of each kind?

8. All round the floor of a room, which is 28 ft. long and 22 ft. wide, there is a border of 2 ft. wide which is left uncarpeted. Find the cost of staining the border at 1s. $1\frac{1}{2}$d. a square yard. Find also the number of yards of carpet 27 inches wide required for covering the rest of the floor, and the cost of this carpet at 3s. 9d. per yard.

9. The diagonal of a rectangular plot is 53 feet, its length is 45 feet; find the side of a square that shall equal this plot in area.

10. How many yards of fencing will be required to enclose a square paddock of 40 acres, and what will be its cost at £2. 12s. 6d. per dozen yards?

11. A courtyard in the shape of an isosceles triangle has each of its equal sides 85 feet in length and its base 154 feet. Find the cost of paving at 2s. 9d. per square yard.

12. The cost of levelling a square piece of ground at 7½d. per square foot is £190. 2s. 6d. Find the cost of surrounding it with an iron fence at 2s. 7d. per yard.

13. The length of a rectangular field is to its breadth as 5 to 2, and its area is 11094 square metres. Find the cost of surrounding it with a fence at 2·25 francs per metre length of boundary.

14. A room is 18 ft. long, 12 ft. wide and 11 ft. high: what length of paper, a yard wide, would be required to line its four walls and ceiling?

15. The length of a rectangular field is 156·5 metres and its width 125·75 metres. If the price of the land is 5·5 francs per square metre and the cost of enclosing the field by a fence is 2·75 francs per metre, what is the ratio of the cost of the field to that of the fence?

16. Explain why there are nine square feet in a square yard.

Given that a metre is 39·37 inches, find the length of a side of the square whose area is 11300 square metres, (*a*) in metres, (*b*) in yards.

17. The floor of a room is 18 ft. long and 14 ft. wide. What will it cost to cover it with linoleum at 2s. 3d. per square yard?

18. A rectangular field is 154·5 metres long by 103·75 metres wide; find its value at 2·73 francs per square metre.

19. If the cost of putting a fence round the field in the preceding exercise were 2·73 francs per metre, show that the ratio of the cost of the fence to the price of the field is very nearly 1 to 31.

20. Find the area of a triangle the sides of which are 3·15 ft., 3·48 ft. and 4·29 ft. respectively. Find the length of the perpendicular let fall from the opposite vertex on the side 4·29 ft.

21. The sides of a triangular field are measured and found to be 209 yards, 276 yards and 115 yards respectively. Find the area.

It is found on examination that the scale used is 0·075 inches too long in each yard. What is the true area?

22. The parallel sides of a trapezium are 97 yds. and 143 yds. respectively. Find the area if the shortest distance between them is 96 yds.

23. The two parallel sides of a trapezium are 650 and 925 yds. respectively. If the area of the trapezium is 661500 sq. yds., find the perpendicular distance between the sides.

24. Find the area of a hexagon, length of side 2 ft.

ANSWERS.

Exercises I., p. 5.

1 29   3 3   4 60   5 53   6 179   7 2376   8. 19.
9. 2134   10 1507   11 252   12 149   13 47.   14 325.
15. 137.   16 177   17 683   18 685   19 57   20 21.
21. 2003.   22. 9   23 729   24 144   25 119.   26 36.

Exercises II., p. 8.

1. 120.   2 13728.   3. 6930   4. 459000   5. 2970.
6. 3264.   7. 1729728   8 660   9 1155.   10. 6105.
11. 43029.   12 128095   13. 253525801   14. 16380.
15. 42504.   16. 252 ; 1081080   17. 181   18. £2.12s.6d.
19. 156035880.   20 18480   21 4950.
22. 10710   23 149 . 1317160   24 3 ; 6027.
25. 472787.   26 2137.   27. 180 ; 18.

Exercises III., p. 13.

1. $5\frac{1}{4}$.   2 $1\frac{23}{24}$.   3 $8\frac{11}{16}$.   4 $1\frac{101}{108}$ 6. 0.
7. 221.   8. $\frac{1}{2}$.   9 264.   10 $\frac{11}{32}$, $\frac{15}{16}$, $\frac{7}{8}$, $\frac{3}{4}$.
11. $10\frac{287}{360}$.   12. $1\frac{2}{15}$ 13 $1\frac{3}{16}$ 14 9 [illegible]   15. $8\frac{347}{1700}$.
16. $14\frac{9}{10}$.   17. $6\frac{7}{18}$.   18 10 [illegible].   19 $11\frac{581}{660}$.   20. $7\frac{9}{28}$.
21. $6\frac{487}{720}$.   22. $7\frac{197}{208}$.   23. $\frac{2}{35}$.   24. $9\frac{3}{16}$ 25. $1\frac{13}{14}$.
26. $\frac{89}{280}$.   27. $\frac{1163}{1470}$.   28. $\frac{1}{2}$ 29. 2.   30. 1.   31. $\frac{5}{9}$.

Exercises IV., p. 17.

2. $4\frac{6}{31}$.   3. $\frac{2}{5}$; 20.   4. $341\frac{13}{52}$.   5. $\frac{39}{50}$.   6. $\frac{19}{24}$.
7. (i) 1 ; (ii) $\frac{1}{4}$.   8. $2\frac{2}{5}$.   9 1   10. 72.   11. $\frac{32}{99}$.
12. 299.   13. 9.   14. 7.   15. 6.   16. 56.   17. 4.

18. $\frac{3}{5}$. 19. $2\frac{97}{180}$. 20. $1\frac{73}{144}$. 21. (i) $\frac{3}{16}$, (ii) $1\frac{4}{7}$.

22. $22\frac{5}{7}$. 23. $\frac{121}{756}$. 24. $30\frac{13}{15}$. 25. $13\frac{1}{14}$. 26. $\frac{65}{333}$. 27. $2\frac{1}{5}$.

28. $\frac{85}{336}$. 29. 1. 30. 1. 31. $2\frac{65}{72}$. 32. $\frac{5}{18}$. 33. 960.

Miscellaneous Exercises V., p. 19.

1. $7\frac{121}{240}$. 2. $\frac{21}{128}$. 3. $38\frac{8}{9}$. 4. $3\frac{13}{108}$. 5. $63\frac{1618}{1659}$.

6. $\frac{833}{1107}$. 7. $\frac{115}{209}$. 8. $7\frac{1}{2}$. 9. $\frac{5}{2464}$. 10. $\frac{5}{12}$.

11. $\frac{77}{144}$. 12. $3\frac{4}{9}$. 13. $\frac{29}{142}$. 14. $2\frac{4}{35}$. 15. $\frac{969}{29393}$.

16. $\frac{17}{48}$. 17. $9\frac{3}{7}$. 18. $\frac{178}{333}$. 19. $4\frac{3}{4}$. 20. $\frac{16}{21}$. 21. 1.

22. $3\frac{1}{2}$. 23. $\frac{68}{45}$. 24. 18s. 9d. 25 $6\frac{3}{4}$. 26. (i) $1\frac{22}{35}$; (ii) $\frac{2}{3}$.

27. $7\frac{311}{952}$. 28. £15,400; £2200. 29. $14\frac{2}{5}$ minutes. 30. £603. 5s. 0d

31. £4. 4s. 11d. 32. $3\frac{1}{3}$ days. 33. 2 hrs. 24 mins. 34. $1\frac{3}{7}$ hours.

35. £45. 36. $\frac{8}{15}$. 37. 3s. 6d. 38. (i) 2; (ii) $6\frac{2}{69}$. 39. 8s. 4d.

40. $\frac{1}{3}$ 41. £1. 10s. 6d. 42 (a) 1 lb. 12 oz., (b) 10 lb. 8 oz.

43. $1\frac{1}{4}$ ac. 44. $\frac{3}{8}$, 19s. 4d. 45. 3s. $1\frac{1}{3}$d. 46. 1·7.

Exercises VI., p. 23.

1. 210·99736. 2. 197·19546. 3. 370·52693. 4. 365·2231.

5. 82·005102. 6. 251·339736. 7. 1035·691. 8. 219·36514.

9. 370·228427. 10. 225·16774. 11. ·533901. 12. 332·72973.

13. ·932402. 14. 8803·96427. 15. 12·7834075. 16. 8·828865.

17. 44·54408. 18. 8·03898. 19. 35·95277.

20. 314·5386162. 21. 910156·00254321. 22. 167·27853383.

23. 556·164495. 24. 547·12625. 25. 4674·162. 26. 301·6252.

27. 32·04147. 28. 72·6781. 29. 148·64236. 30. 64·0888.

31. 4639·55663. 32. 59·73286. 33. 18·903296.

Exercises VII., p. 26.

2. ·06424. 3. ·0000128. 4. ·005025. 5. 366·2348.

6. ·0699. 7. ·109733. 8. ·011214. 9. 8·74894.

10. 43·2457. 11. 3·2219. 12. 808·5363. 13. ·0002194.

14. 340·1466. 15. 2101·127. 16. 79682·44. 17. ·012016.

18. ·10875. 19. ·167433. 20. ·011214. 21. ·107151.

22. ·0001287. 23. ·0017283. 24. ·011836.

Exercises VIII., p. 28.

1. ·0016. 2. ·22. 3. 100·37. 4. 28920. 5. ·0051.

6. 1562·5. 7. 4·29. 8. 66600. 9. 3·7356. 10. ·0117.

11. 76·923. 12. 1·2845. 13. ·401. 14. 2·49. 15. ·435.

16. ·314. 17. 12000. 18. ·00334. 19. ·02496. 20. ·4007.
21. 2501. 22. 98·9. 23. ·0245. 24. 325. 25. 4632700.
26. ·1883. 27. 40·9. 28. 34·67. 29. 2783 30. ·0003.
31. 9060. 32. ·005 33. 631, remainder ·033 in. 34. 2·8.
35. 9675 ; ·005. 36. 275 ; ·003.

Miscellaneous Exercises IX., p. 28.

1. 4·92. 2. 4·255. 3 1005·6079 4 ·097485 ; ·25.
5. ·02578 ; ·009216. 6 3·059 7. 1·0505 ; 9·8. 8. 401 ; 2·49.
9. 9740·874 ; 1813·68 10. 440·82. 11. ·09375 ; ·78125.
12. (i) 76·9, (ii) ·06 13. (i) 290, (ii) 6·2082 14 (i) 1217·605, (ii) 45.
15. ·258 , 4·05 ; 1·045. 16. (i) ·097485, (ii) ·25 17. 2. 18. 80.
19. ·08. 20 ·0481. 21. ·0209. 22. ·009216. 23. $\frac{123}{256}$.

Exercises X., p. 32.

1. 2010·114. 2 273·9246 3 1704·635 4. 105·0567.
5. 6·020297. 6. 302·203 7 ·00117297 8. 16·894668.
9. 26·3896. 10. 44·1922. 11 6·47522 12. 169·209.
13. ·43051. 14. 1826·392. 15 ·037274 16. ·00864.
17. 1274·897. 18 1·8806. 19 197905·8578 20 714·1791.
21. 47975·1. 22. ·016558. 23 244393·4. 24 51·258.
25. ·01661. 26. 20·7969 27 ·052583 28 232·0346.
29. 3673·706. 30. 324·0642. 31 36·074. 32. 14530·625.
33. ·032370.

Exercises XI., p. 33.

1. 32046. 2 29·5. 3 ·00825 4 119·57.
5. 6·725. 6. ·059106. 7. 43·25. 8. ·005107.
9. ·070885. 10. 31·012. 11. ·00175 12 4230.
13. 32·79. 14. 435. 15 381·8. 16. ·03764.
17. 317·5. 18. ·1372. 19. 717·8. 20 4073.
21. ·04705. 22. 555. 23. ·00243. 24. ·245.
25. ·628. 26. ·005459. 27. ·001609. 28. 2460.
29. ·00348. 30. 13·75. 31 130. 32 19880.
33. ·03. 34. ·007793. 35. 90·16. 36. 3·921. 37. ·03674.

Exercises XII., p. 36.

1. ·583. 3. 2·59375. 4 ·9375 ; ·028. 5. 1·09375.
6. 11·8288. 7. ·9412. 8. 11·7578. 9. 18·402969.
10. $\frac{73}{200}$; $\frac{1}{8}$; $\frac{7}{2000}$; $\frac{403}{80}$; $\frac{35}{4}$. 11. $8\frac{176}{661}$. 12. ·5269.

13. ·1875. **14.** ·189. **15.** 1·271. **16.** $\frac{61}{495}$; $11\frac{611}{1350}$; $\frac{17267}{4995000}$.
17. $\cdot 003\dot{6}$. **18.** $1\cdot 2967\dot{8}$; $\cdot 63\dot{0}1\dot{2}$; ·32115; $2\cdot 89\dot{6}3\dot{6}$. **19.** $41\cdot 24\dot{6}\dot{3}$.
20. $\frac{161}{300}$. **21.** $\frac{1}{8}$. **22.** ·2. **23.** $\cdot 0\dot{3}$. **24.** $\frac{1}{5}$; $\frac{19}{13}$. **25.** $19\cdot\dot{0}\dot{9}$.

Exercises XIII, p. 38.

1. 189. **2.** $\cdot 347\dot{2}$. **3** 8·304 pence. **4.** 2026·248 minutes.
5. 37·072 lbs. **6.** 4621·32 ft. **7.** 8s. $7\frac{3}{4}$d.
8. ·75. **9.** ·325. **10.** £22. **11.** 54701·425.
12. 663·18. **13.** $\frac{677}{8750}$. **14.** 5·4198. **15.** 78·012. **16.** 6d.
17. ·3140625. **18.** 2·7765625; 2 qrs. 7 lbs.

Miscellaneous Exercises XIV., p. 39.

1. 7256. **2.** £4. 3s. $1\frac{1}{4}$d. **3.** 6·232 **4.** ·000008.
5. ·00161. **6.** ·826. **7.** $43\frac{1}{3}$ **8.** $\cdot 5\dot{4}$; $\cdot 5\dot{3}$; $\cdot 01\dot{2}$.
9. 1·044. **10** 1·6 **11.** ·856 **12** 15; 11.
13. 1·82496; $\frac{15449}{3700}$. **14** $\frac{49}{55}$; 420 **15** $4\cdot 81\dot{3}$.
16. $\frac{1}{4}$. **17** 1 : 1. **18** ·00681 **19.** $19\cdot\dot{0}\dot{9}$.
20. $\cdot\dot{2}7\dot{0}$; $\frac{859}{49500}$. **21.** $\frac{71}{105}$. **22** $6\cdot 7\dot{6}$. **23.** $\cdot 4\dot{6}$.
24. 1·08448. **25.** ·9765625; 1·024; 1. **26.** ·975 **27.** $\cdot 2\dot{3}$.
28. $19\cdot\dot{0}\dot{9}$. **29.** 15. **30.** ·527. **31.** 3·1097. **32.** ·005469 in.
33. 38·64 lbs **34.** 206·58 min. **35** 2 cwt. 1 qr. 12 lbs.; 0·066.
36. £1. 19s 6d **37** £11. 14s. 8d. **38.** 2s. 4·3d

Exercises XV., p. 44.

1. ·0009. **2.** ·000000169. **3.** ·0004 **4.** ·18. **5.** $\frac{1}{256000}$.
6. 39·0625; 244·141; 3906·25; 244140·6.
7. 8·7025; 25·67; 11·22; 37·595. **8.** 7·25. **9.** ·000271. **10.** 1.
11. ·003639.

Exercises XVI., p. 48.

1. 729. **2.** 327; 28·349. **3.** ·437962 **4.** 115·23. **5.** ·0125.
6. 7006; $9\frac{1}{2}$. **7.** 9·539. **8.** 709. **9.** 70·205; $\frac{13}{15}$. **10.** ·02604.
11. ·186. **12.** ·2317. **13.** 8·426. **14.** 9·1109. **15.** 7070. **16.** 22·004.
17. 10·001. **18.** 3007. **19.** $\frac{23}{48}$; ·2846. **20.** (i) 11·1108; (ii) 15·3011.
21. 57·004. **22.** 5·07. **23.** (i) 2·828; (ii) ·014. **24.** 141·05. **25.** 62573.
26. $6\frac{7}{18}$. **27.** 7072. **28.** 4·168. **29.** 1·046. **30.** (i) 4·5; (ii) $7\frac{2}{3}$.
31. $5\frac{2}{3}$. **32.** (i) 0·73; (ii) 0·9; (iii) 1·2; (iv) 1·47. **33.** 20·46.
34. 0·5. **35.** 0·73. **36.** $253\frac{1}{3}$.

Exercises XVII., p. 51.

1. 2·06215. **2.** $47\frac{5}{11}$ miles per hour.
3. $30\frac{51}{88}$ miles per hour. **4.** 3 lbs. 6 oz.

5. 50 miles per hour. 6. 26722 36. 7. 26·27.
8. 8s. $1\frac{1}{2}$d. 9. 57·29925 ; ·6295. 10 £1672. 2s. 6d.
11. $5356\frac{13}{30}$; $15\frac{771}{5900}$. 12. 64 05 feet. 13. $149\frac{11}{48}$.
14. 30·5 ft. per sec. 15. 92·9 lbs 16. 59·99. 17. $6\frac{3}{4}$ mins.
18. 13 st. 4 lbs. 19. 887040. 20. 70400.

Exercises XVIII., p. 56.

1. ·00275. 2 70 ; 9 3 4 32d 4. £8 11s. $10\frac{1}{2}$d.
5 10. 6 £481 5s. 7. ·0859 8 $\frac{70}{99}$.
9. 485 miles. 10. $7\frac{1}{3}$ 11 £224, £240, £350.
12. 22 cwts 2 qrs. 13. 6524 14. $59\frac{1}{2}$, 68, $76\frac{1}{2}$ 15. 5.
16. $\frac{27}{86}$; $\frac{285}{184}$. 17. £7 ; £11 13s. 4d , £16. 6s. 8d. ; £21.
18. £7173. 6s. 8d. ; £8070 ; £8608 ; £8986 13s. 4d.
19. 6s. 6d. 20 £4 5s. ; £3 5s. ; £2 10s.

Exercises XIX., p. 59.

1. £2. 5s 2 125. 3. £9 1s 7·2d 4 $5\frac{7}{8}$.
5. 18·5 %. 6. $28\frac{4}{7}$. 7. £875 ; $7\frac{11}{17}$ 8. 142800.
9. $12\frac{1}{2}$. 10. $26\frac{2}{3}$. 11 £446 10s 12 £1 19s 4d. ; 21·4 %.
13. Sulphur 28 lbs , charcoal 42 lbs , nitre 210 lbs 14. 12·72.
15. 5 per cent. 16. 20 per cent 17 £62. 10s 18. 2 ; 14.
19. 19 per cent. 20 19 26 21 £196 10s. 22. 20200.
23. 5s. 3d. 24. 7·95 ; 16 5 25 $12\frac{1}{2}$d 26. 67, 26·3, 6 7.
27. (a) 112 lbs., (b) 50. 28 98 56 lbs., 2·24 98 56. 29. £39. 10s.

Exercises XX., p. 66.

2. $x+\frac{x}{2}+\frac{x}{4}$, or $x+x^2+x^3$. 3 120 ; 100 ; 12.
4. 96 ; 432 ; 396. 5 93 6 -44 7. 156.
8. 20. 9 2. 10 45 11. 9. 12. 5.
13. 1640. 14 $\frac{1}{4}$. 15 -1560 16. 14179 7.

Exercises XXI., p. 67.

1. $11(a+b+c+d)$. 2. $13a-13b+9c+2d$. 3. $10a$.
4. $23x-5b$. 5. $6x-6y+8z+a$. 6. $ax+2by+2s-4z$.
7. $7xy+5az-bc+5mn-3p$ 8 $2a^2+3a+3b-c$.
9. $47a^2-b^2+3c^2$. 10. $15ab-7cx+4ek-8$.
11. $12a^2+6ab-b^2+3a+4b$, 31. 12. 194. 13. 109.
14. $2a^2b+7ab^2+3b^3$. 15. $9a^3+3ac^2-6a^2c+3ade-7acd$.

Exercises XXII., p. 69.

2. $6a^2-4a+3b-4$. 3. y^2+y-4a. 4. $3a^2-7b^2$. 5. $2xy+16+5$.
6. $5x^2-2xy-5$. 7. $-a^3-10a^2b-ab^2$. 8. $6a^2b+2b^3$. 9. $5a+8b-8c$.
10. $-ax-7by+2cz+2$. 11. $3x^3-3x^2+3x-3$. 12. $2a+2b+9c$.
13. $3z+5y-y^2-2x+30$. 14. $10mz+4bx-10ay+1$.
15. $a^3-7a^2b+6ab^2+4b^3$. 16. $\frac{59}{40}y-\frac{x}{12}$. 17. $6ab-8b^2$.
18. $14x^2y^2-9y^4$; $4x^4+4x^2y^2-\frac{1}{4}y^4$. 19. $x^3-x^2y+7xy^2-5y^3$.

Miscellaneous Exercises XXIII., p. 70.

1. $3ac+4y-z$. 2. $ax+6by-11cz+mn$. 3. 9. 4. 16. 5. 22·41.
6. 2. 7. -6. 8. 36. 9. (i) $-4·6$; (ii) 10; (iii) $-\frac{11}{10}$; (iv) $-\frac{18}{25}$.
10. (i) $-40\frac{1}{2}$; (ii) $1\frac{37}{39}$; (iii) $-13\frac{3}{4}$. 11. $a^2-2ax+5x^2$.
12. $-7a+13b+5c$; 34. 13. $-12xy$; -24. 14. x^3+13x^2+32; 64·625.
15. $-2ax$; -18. 16. $2a-10b+5c$; $-3·7$. 17. $\frac{b^3-bc^2+2c}{b^2-c^2}$, $-\frac{1}{3}$
18. (a) $a-b$, (b) $a+b$, (c) $\frac{a-c}{a}$. 19. 50. 20. 20.
21. $\frac{ax^3+4x^3+3+bx}{x^4}$. 22. 1·75. 23. 20, 2·5, 2·4, 26·67.
24. $\frac{a}{5}-\frac{5}{12}b$.

Exercises XXIV., p. 74.

1. $a^6-a^5b+9ab^5$. 2. $x^3-9x^2y+26xy^2-30y^3$.
3. $x^3-9x^2y+26xy^2-24y^3$. 5. $16a^4-72a^2b^2+81b^4$.
6. $20x^9-22x^7+35x^5-17x^3+5x$; -21. 7. $a^3+b^3+c^3-3abc$.
9. $35x^4-12x^3y-12x^2y^2+72xy^3-35y^4$. 10. $a^3-b^3+3a^2b$.
11. $x^3+xyz-y^3+z^3$ 12. x^6-a^6.
13. $a^4-2a^2b^2+b^4+4abc^2-c^4$. 14. $x^3+y^3+z^3-3xyz$.
15. $8a^4+4a^3b-54a^2b^2+27ab^3+27b^4$. 16. $\frac{9x^4}{2}+2x^2+\frac{8}{9}$.
17. $4a^3+8a^2-13ab^2+4ab+5a-6b^3-5b^2+2b+1$.
18. $x^7-61x^5+167x^4-108x^3+61x^2-168x+108$.
19. $a^4-3a^3b-9a^2b^2+23ab^3-12b^4$. 20. $x^5-4a^2x^3+3a^4x-a^5$.

Exercises XXV., p. 76.

1. $x+y$. 2. $a^2+4ax+x^2$. 3. x^2-6x+9.
4. a^2+ab+b^2. 5. a^2-ab+b^2. 6. $a^2-3ax+x^2$.
7. a^2-ab-b^2. 8. $a^2+2ab+2b^2$.
9. $x^4+2ax^3+3a^2x^2+2a^3x+a^4$. 10. $a^2+ab-ac-bc$.
11. $x^4-ax^3-6a^2x^2-7a^3x+49a^4$. 12. $3x^2-2xy+3y^2$.

13. $x^4 + ax^3 - a^3x - a^4$.
14. $x^2 + y^2 + z^2 - yz - xz - xy$.
15. $x^3 - 5x^2y + 6xy^2 - y^3$.
16. $x^4 + p^2x^2 - pqx + q^2$
17. $5xy(x + y)$.
18. $x^3 - 3x^2 + 11x - 8$.
19. $x^4 + ax^3 - a^3x - a^4$.
20. $x^3 - 2x^2 + 4x + 7$.
21. $2x^2 + xy + 3y^2$.
22. $1 - 2x + 3x^2 - 3x^3 + 3x^4 - 3x^5$
23. $3x^2 - 2abx - 2a^2b^2$.
24. $a^2 - b^2$.
25. $7x^2 + 5xy + 2y^2$.

Exercises XXVI., p. 78.

3. $-27a - 3b$
4. $-10x + 12y$.
5. $13a - 21b$.
6. x.
7. $x + 2$.
8. $a - y$
9. $2a - b - d$.
10. $c + 14b - a$.
11. 0
12. $-7b$.
13. $2a$.
14. $3(a + 3b)$.
15. $10\frac{2}{3}$.
16. (i) 1 : (ii) $\frac{101}{4}$.
17. $3a + 8b$.
18. $a + 5b - 4c$.
19. b
20. $2a - 13b + 10c$.
21. $35a - 27b$.
22. 128.
23. $\frac{1}{a}$
24. $125\frac{7}{12}$.
25. (i) $\frac{2}{3}$, (ii) $15\frac{5}{11}$, (iii) -15, (iv) $3\frac{1}{7}$.

Miscellaneous Exercises XXVII., p. 79.

1. $6x^5 - 25ax^4 - 10a^2x^3 + 17a^3x^2 + 28a^4x - 4a^5$
2. $x^4 - 2x^3 + 3x^2 - 2x + 1$.
4. $x^2 + 2x + 3$
5. $2x - 30y + 53z$.
6. (i) $x^3 - 9x^2y + 26xy^2 - 24y^3$, (ii) $4x^2 + 5xy + 10y^2$
7. (i) $\frac{35}{192}$, (ii) $9\frac{11}{4}$
8. 0
9. $3a^2 - 5ab - ac - 3ac^2 - ab^2c + 5bc + bc^2 + b^2c^2$; -2.
10. $367a + 6b + 690c - 240bc - 120ab$; 2.
11. (i) $\frac{5x^3}{6} - \frac{x^2y}{6} + \frac{xy^2}{6} + \frac{y^3}{6}$, (ii) $2a^3 - 3a^2b$.
12. $-2x - 30y + 58z$.
13. $2b^3 - 3b^2c$.
14. $-35x + 18y + 17z$.
15. $b^2 - a^2 + \frac{b^4}{a^2} - \frac{a^4}{b^2}$.
16. $\frac{x^5}{4} - \frac{4x^4}{3} + \frac{77x^3}{24} - \frac{43x^2}{12} - \frac{11x}{4} + 9$.
17. $2b(x + y)$.
18. $\frac{x^3}{6} + \frac{x^2y}{3} - \frac{xy^2}{3} + \frac{5y^3}{6}$.
19. $2(x - y)$.
20. $\frac{8a^2}{5} - \frac{5b^2}{4} + c^2 - \frac{27d^2}{7}$.
21. $a(c - b)$.
22. $b(a - c) + 3ac$.
23. $\frac{a}{3} + b$.
24. $3x^2$.
25. 4.
26. -1.
27. $\frac{1}{8}$.
28. $\frac{5}{8}$.
29. -22.
30. $-\frac{1}{7}$.
31. $-\frac{1}{2}$; $\frac{1}{4}$; $-\frac{1}{4}$; $3\frac{1}{4}$.
32. $-4a^2x^2y$.
33. (i) $2a + y + x$; (ii) $\frac{6}{25}$.
34. $2x^3 - a^3 - 2a^2 + 2a + 7$
35. $\frac{1}{2}$.
36. $\sqrt{\left(\frac{a^2}{b^2} + 1\right)} \times \frac{w}{n^2}$.
37. $\frac{(x^2y^3 - xy^4)^{\frac{1}{2}}}{\sqrt{(x + y)}}$.
38. $\sqrt{(x^3y^3 - x^2y^4)} \times (5x + 3y)$.
39. $5a^3 - 10a^2b + 5ab^2$.
40. $4a$, $4a^2 + 4ab$.
41. $3a^2 - 2ab + b^2$.
42. $3x + 4y$.

Exercises XXVIII., p. 87.

1. 7. **2.** 12. **3.** 9. **4.** $4b+2$. **5.** 12. **6.** 10.
7. 5. **8.** 15. **9.** $4\frac{1}{8}$. **10.** 20. **11.** 11. **12.** 12.
13. 13. **14.** 14. **15.** 1. **16.** $4\frac{3}{8}$. **17.** 4. **18.** $\frac{40}{21}$.
19. 13. **20.** 6. **21.** 4. **22.** 3. **23.** 4. **24.** $\frac{3}{4}$.
25. -5. **26.** 7. **27.** (i) 5, (ii) 172. **28.** 25·3. **29.** 200.
30. (i) 11, (ii) 36, (iii) 4. **31.** 450. **32.** 12. **33.** 14. **34.** 6.

Exercises XXIX., p. 90.

1. $8\frac{4}{7}$; $6\frac{3}{7}$. **2.** 24 ft. **3.** $3\frac{3}{4}$. **4.** 100. **5.** 22; 37.
6. 10. **7.** 6. **8.** 3; 9. **9.** 6 **10.** 9000.
11. 180; 205. **12.** A, £800; B, £320. **13.** 42; 6. **14.** £660; £340.
15. 30. **16.** 55. **17.** $76\frac{1}{2}$; $202\frac{1}{2}$. **18.** $1\frac{1}{4}$d. **19.** 23·4; 15·6.
20. £1. 13s. **21.** 37; 21. **22.** 62. **23.** £1. 2s. $10\frac{1}{2}$d. **24.** $2a$.
25. $\frac{22}{15}ab$. **26.** 8·24; 9. **27.** 2·79; 9·77. **28.** 0·793 ft.; 0·528 ft.
29. 4·39; 15·61. **30.** 30; 20. **31.** 44·52 yrs.; 27·84 yrs.
32. (a) 18, 20, 22; (b) 15, 45. **33.** 5s. 6d.; 1s. 6d.
34. $c(c^2-1)$; 336 miles. **35.** £2. 2s.; 14s.

Exercises XXX., p. 101.

1. ·0254; ·3048; ·9144; 5·0291. **2.** 20·1164; 201·1644; 1609·3149.
3. 3 poles 2 chains. **5.** 9 yds. 1 ft. 8·484 in. **6.** 6296·4. **7.** 2414·016.
8. 39·37 inches. **9.** 160·9315. **10.** 5s. 4·32d. **11.** 3076.
12. (i) 3069·04752; 935·45; (ii) 6 miles 369 yds. 1·8 ft. **13.** 8·224.

Exercises XXX. (a), p. 110.

1. 10 cm., 5 cm. **3.** 97·2 cm. **4.** 2·45 in.
5. 84° 48′, 57° 24′, 37° 48′; 13 in.
6. $2\frac{3}{8}$ in., 4 in., $5\frac{1}{4}$ in; 28° 56′, 46° 36′, 104° 28′.

Exercises XXXI., p. 116.

1. (i) ·92899; (ii) 8·36097. **2.** 1011·67. **3.** 836·097. **4.** ·2019.
5. ·645137. **6.** 10. **7.** 36440497. **8.** Latter; ·03.

Exercises XXXII., p. 126.

1. (i) 208·6 ft.; (ii) $\frac{5}{11}$. **2.** 233·08 yds. **3.** 622·25; 155·56.
4. $13\frac{1}{7}$ ft. **5.** 488·87 ft. **6.** 60 ft. **7.** 250 sq. ft.
8. 600 sq. ft. 36 sq. in. **9.** 8·66 ft. **10.** $149\frac{2}{3}$ yds.; £33. 11s. 6d.
11. 3 ft. 9 in. **12.** £1464. 2s. **13.** £91. 16s. 2d. **14.** £23. 0s. 4·8d.
15. £1. 15s. $9\frac{5}{7}$d. **16.** $87\frac{1}{3}$ yds.; £9. 16s. $10\frac{1}{2}$d. **17.** £1. 13s.

18. $52\frac{1}{4}$ sq. ft. **19.** 16 **20.** $112\frac{1}{2}$. **21.** £17. 3s. 4d. **22** £2. 0s. 11d.
23. 10 ft. **24** £180 **25** £24 10s. **26.** 1 r. 10·233 po. **27** $44\frac{4}{25}$ po.
28. 8·68 mins **29.** £3 5s 10d **30** 217·9 yds. **31.** 6s. 3d.
32. £13 3s $6\frac{1}{2}$d **33** 14 ft 3 in **34.** 2377·292 ft **35.** 18 yds.
36. £68 17s 9·6d **37** 32·5, 148; $98\frac{2}{3}$, $153\frac{1}{3}$ sq. yds.
38 $\frac{1}{2}$ mile; 10d **39** £108 4s 4·8d; £10 11s 7d **40** £1 10s. $7\frac{1}{2}$d.
41 418 yds. **42** (a) 6 in. 2 lbs.; (b) $\frac{ab}{12}$s.; (c) 672 sq. ft., 5s. 5d.
43. £146 4s **44** 9 yds.

Exercises XXXIII., p. 132.

1 618·6 sq. ft **2** 15 acres **3** 84 sq. ft.
4 210 sq. in **5** £3 6s **6** 60 sq. yds
7 2390 sq. yds **8** 3 ac 1 r **9** $133\frac{1}{3}$ sq. yds.
10 61 ac 2 r 1·6 per **11** 12 ac 2 r 11·7 po **12** £5 10s
13 £8 13s 6·75d **14** 3000 sq. ft **15** 150, 200, 250, 15000 sq. yds.
16. 270 sq. ft **17** 109·81 sq. ft **18** 42 ac 1·6 po.
19 43·3 sq. ft **20** 3 ac 3 r 35 po 6 ch $13\frac{1}{2}$ yds **21** 25 chains.
22. 99·82 sq. ft **23** 38 ft **24** 24 ch **25.** 1·46 ac.

Exercises XXXIV., p. 136.

1. 15548 sq. ft **2** 1140800 acres **3.** 6 ac 3 r
4. 1 ac 2 r. **5** 760375 sq. links **6** 1428 sq. ft
7. 34840 sq. yds **8** 44 sq. ft **9** 839·5 sq. ft.
10 408·7 sq. ft **11** 31 ac 3 r 16 po **12** 4950
13 3 ac 1 r 9·2 po **14** 3 ac 4 r **15.** 23, 27 **16** 6·92 ac

Exercises XXXV., p. 139.

1. 20 inches **2** 300 sq. yds **3** 258 lb.
4. 374·122 sq. ft **5** 1039·23 sq. ft **6** 10 ft 6 in.
7 200 links **8** 36 yards

Miscellaneous Exercises XXXVI., p. 141.

1. $\frac{3}{4}$ sq. in **2** £41. 14s 4·8d **3.** 5s. $0\frac{1}{4}$d **4** £79. 19s. 7·8d.
5. $\frac{5}{863}$ ac. **6** 486; £4 1s 0d. **7** 45 yds. **8.** 3146 sq. yds.
9. 1·003 ft. **10.** 52 sq. ft **11** 71 yds. 4 in **12.** 10.
13. 24 ft 9 in. **14.** 1080 **15.** 68·58. **16.** £2. 8s. 3·8d. **17.** 1s. 3d.
18. 6 yds. **19.** 220 yds.; 55 yds. **20** £3. 8s 3d. **21.** $97\frac{1}{2}$ yds.
22. 12 min. 4 sec **23** £22 13s 2·2d. **24.** 22 acres.
25. 20·78 ft **26.** £33 **27.** 844.
28. 355680 sq. links. **29.** 152·075 sq. chains. **30.** £105.

MISCELLANEOUS EXERCISES.

Section I. Vulgar Fractions, p. 143.

2. $1\frac{1}{2}$. **3.** $\frac{43}{120}$. **4.** $\frac{7}{15}$. **5.** $\frac{29}{58}$. **6.** £1284. 5s.

7. $\frac{1}{7}$. **8** £1. 12s. 4d., £2. 8s. 6d., etc. **9.** $\frac{7}{24}$.

10. 200. **11.** £3. 5s. **12.** 2116, 10360. **13.** (i) $\frac{11}{28}$, (ii) $\frac{7}{8}$.

14. 1s. 8d. **15.** *A*'s, £115; *B*'s, £172. 10s.; *C*'s, £230; *D*'s, £276.

16. *C*'s is $\frac{91}{99}$ of *B*'s; 200 . 99 . 91. **17.** Latter. **18.** £18. 16s. 3d

19. Nitre, 107·24 lbs.; sulphur, 21·28 lbs.; charcoal, 32·48 lbs.

Section II. Decimal Fractions, p. 144.

1. 355·46905. **2.** 70·1307481. **3.** 406·449933. **4.** 387·32926401.

5. 203·66519. **6.** 558·236334. **7.** 1·222231. **8.** 232·4335.

9. 0·909100. **10.** 94·9700881. **11.** 60·108375. **12.** 8·98079072.

13. 0·03106. **14.** 0·05258. **15.** 1·558. **16.** 62·25.

17. 284·7. **18.** 31·06. **19.** 3·123. **20.** 0·006223.

21. 0·03106. **22** 373·3. **23.** 0·007529. **24.** 0·02665.

25. 0·4055. **26.** 2817. **27.** 0·3733. **28.** 1704.

29. 0·002466. **30** 373·2. **31.** (i) 0·3123, (ii) 1706.

32. (i) 3·123, (ii) 1706. **33.** (i) 62·25, (ii) 0·4055. **34.** 6·6363.

35. $7\frac{1}{8}$. **36.** $\frac{17}{20}$. **37.** 24. **38.** 0·3391. **39.** 0·8125.

40. 105·9 lbs. **41.** 0·5. **42.** 2·52. **43.** 4358 lbs. **44.** 9·36d.

45. $\frac{309}{113}$, 0·2727. **46.** 0·004547. **47.** 13·26. **48.** 3·7725.

Section III. Involution and Evolution, p. 146.

1. 9·345; 1·748.

2. (i) 2·886; (ii) 0·3795; (iii) 0·7142; (iv) 0·226; (v) 0·756.

3. 4·168. **4.** 7·25. **5.** 91·5416. **6.** 10·237.

7. 2·00025. **8.** 0·00025. **9.** 144 sq. yds. **10.** 916·3 yds.

11. 5 ft. **12.** 29·58 ft. **13.** 57 yds. **14.** 701 yds.

Section IV. Averages and Ratio, p. 146.

1. $2133\frac{1}{3}$. **3.** (*a*) $\frac{27}{58}$; (*b*) $\frac{285}{184}$. **4.** 28·87.

5. $59\frac{1}{3}$, 68, $76\frac{1}{2}$. **6.** $\frac{2}{5}$, $\frac{19}{47}$, $\frac{4}{123}$. **7.** 1587·32 lbs.; 720 kilos.

Section V. Percentages, p. 147.

1. $4\frac{1}{2}\frac{6}{1}$, or 4·76 per cent. 2. 71·75. 3. 31·4 per cent.
4. 903·1 cub. ft. 5. £2985, £985. 6. 42 lbs.
7. £224, £240, £350, 27·4 %; 20·48 %, 43 %.
8. Sulphur, 38 lbs.; charcoal, 58 lbs; nitre, 304 lbs.
9. £640, £720, £768, £800; 21·68, 24·59, 26·23, 27·32.
10. 21, $22\frac{1}{4}$, 8·3. 11. £2 6s 3d. 12. 20·2. 13. $\frac{6}{13}$.
14. £1126 $\frac{3}{7}\frac{4}{1}$ stock; £28. 3s 4·5d; £33 13s 0·9d.
15. £26 13s.
16. £7, £11. 13s 4d, £16 6s 8d, £21; 12·5 %, 20·83 %, 29·17 %, 37·5 %. 17. £62 10s.

Section VI., p. 148.

1. 1s. 2. 33 nap. 0·48 francs. 3. 94·58.
4. 14·81 oz. 502·3 grams. 5. 18s 11d. 6. 691·32 lbs; 313·58 kilos.

Section VII. Algebra, p. 149.

1. [illegible] $x+y$ [illegible] 2. $\left(\frac{2x^2 \quad y^2z}{x-y^{\frac{1}{2}}}+4\right)^{1}$. 3. $\sqrt{\left(\frac{a^2}{b^2}+1\right)\frac{m}{n^2}}$
4. a b, 5·61. 5. 0·00029. 6. a^8, a^{12}, 256, 4096.
7. $-7a+3b$ c. 9. $3a$. 10. $8xy$.
11. (i) 0, (ii) $\frac{2}{7}$. 12. c. 14. (i) $29\frac{2}{7}$, (ii) $3\frac{1}{12}$, (iii) $3\frac{1}{7}\frac{9}{7}$.
15. (i) 4, (ii) $2\frac{2}{5}$, (iii) 2 [illegible]. 16. (i) $\frac{11}{3}$, (ii) $\frac{15}{2}$, (iii) $\frac{1}{2}$.
17. $\frac{pr}{x^2+px+q}$. 18. x 1, R 5. 19. 2·952.
20. 14·65, 11·08, 0·2436. 21. 21·13. 22. 132 ft. 23. 474·7.

Section VIII. Multiplication and Division, p. 151.

1. $x^5-4x^4y+4x^3y^2+x^2y^3-11xy^4-3y^5$.
2. x^3-3x-4; $(x+1)(x-4)(x$ $3)$.
3. $x^8-2x^6+4x^5-5x^4-4x^3+10x^2-12x+9$. 4. x^6-
5. $x^5-8x^4y+14x^3y^2+9x^2y^3-6xy^4$.
6. $4x^5+7x^3y^2-10x^2y^3+xy^4$ $20y^5$; $-4\cdot0803$. 7. $2a^3$.

Section IX. Simple Equations, p. 151.

1. 4.
2. 2.
3. $\frac{1}{2}$.
4. $-\frac{27}{4}$.
5. $\frac{3}{4}$.
6. 2.
7. $\frac{11}{13}$.
8. $\frac{2(a^2+b^2)}{a+b}$.
9. $\frac{5}{11}$.
10. $\frac{2ab}{a+b}$.
11. $\frac{3a+8b}{5}$.
12. $a+b$.
13. 17.
14. $\frac{1}{2}$.
15. $\frac{1}{2}(a+b+c+d)$.

Section X. Simple Equations, p. 152.

1. £175, £225.
2. £7. 10s., £9, £7. 4s.
3. 20, 24, 17.
4. £2. 7s. 6d.
5. A, £10; B, £10; C, £20.
6. 17.
7. 1s. 10½d., 1s. 8d.
8. A = 36·18 ft., B = 40·2 ft.
9. 57 miles.
10. £150, £250.

Section XI. Mensuration, p. 153.

1. 48000 sq. ft.
2. 1186 francs.
3. 1·29 m., ·86 m., 967·5 francs.
4. 39·06 sq. in.
5. 5565 yds.
6. 17·89 ft.
7. 11,400,000, 17,100,000, 3,800,000.
8. £1. 3s., 64 yds., £12.
9. 35·49 ft.
10. 1760 yds., £38·5.
11. £42. 7s.
12. £13. 8s. 7·2d.
13. 1049 francs.
14. 292 ft.
15. 69·74 · 1.
16. 106·3, 116·25.
17. £3. 3s.
18. 43770·19 francs.
20. 5·405 sq. ft., 2·52 ft.
21. 15870 sq. yds., 13510 sq. yds.
22. 11520 sq. yds.
23. 840 yds.
24. 10·39 sq. ft.

INDEX.

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